This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School
for Boys and Girls Loueizeh - Baabda

Stage Exam 04

20-25JUN2025

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	12M
Teacher :	Bechara Kordahi
Duration :	4 hours
You are to answer the questions to this exam on : Squared paper	

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

This exam is formed of 10 questions each is worth 20 points . Any 8 questions must be answered in full . If you answer more than 8 questions , then the first 8 questions will be considered for correction .

Question I-(20 points)

In the table below , only one of the proposed answers to each question is correct . Write the number of each question and indicate ,**with justification ,** the correct one .

$N°$	Questions	Proposed answers		
		a	b	c
1	The domain of definition of the function $f(x)=Ln\left(e^{x+1}-1\right)$ is	$]0,+\infty[$	$]-1,+\infty[$	$]-\infty,-1[$
2	The limit of $x(1+Lnx)$ as $x\to 0^+$ is	$-\infty$	1	0
3	An asymptote to $f(x)$ defined over $\Re$ as $f(x)=(x+2)e^{1-x}-x-1$ when $x\to+\infty$ is	$y=x+2$	$y=x+1$	$y=-x-1$
4	A box contains 2 Red balls and 5 Black balls . Three balls are selected successively with replacement . The number of ways of selecting at least 1 Red ball is	25	150	218
5	A and B are two independent events where $P(B)=0.4$. Then , $P\left(\overline{B}/A\right)=$	0.4	0.5	0.6

Question II-(20 points)

In the space referred to a direct orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j}\ ,\ \vec{k})$, consider the following points : A (4 ; 0 ; 1) , B(2 ; 1 ; 2) , C(2 ; 0 ; 3) and E(3 ; –1 ; 0).

1) a- Write an equation of the plane (P) determined by A , B and C.
   b- Show that A is the orthogonal projection of E on (P).

2) a- Show that triangle ABC is right-angled .
   b- Calculate the area of the triangle ABC .
   c- Calculate the volume of the tetrahedron EABC .

3) (Q) is the plane with equation x –2y –2z – 2 = 0 .
   Show that (Q) passes through A and is perpendicular to (BE) .

4) a- Write a system of parametric equations of the line (BC) .
   b- Let M be a variable point on (BC). Prove that the distance from M to (Q) remains constant as M moves on (BC).

Question III-(20 points)

Consider the function f defined over $\Re$ by $f(x) = (x+1)^2 e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-2)$.

b- Determine $\lim\limits_{x\to+\infty} f(x)$ and deduce an asymptote to (C).

2) Show that $f'(x) = (1-x^2)e^{-x}$ and set up the table of variations of f.

3) The line (L) with equation y = x intersects (C) at a point with abscissa α.

Verify that $1.4 < \alpha < 1.5$.

4) Draw (L) and (C).

5) Let F be the function defined on $\Re$ by $F(x) = (px^2 + qx + r)\, e^{-x}$.

a- Calculate p, q and r so that F(x) is an anti-derivative of f (x) .

b- Calculate the area of the region bounded by (C), the axis of abscissas and the two lines with equations x = 0 and x = 1.

Question IV-(20 points)

A shop sells two types of earphones E_1 and E_2 and three types of batteries B_1, B_2 and B_3. During the promotion period, some items are placed in two baskets U and V.

Basket U contains 15 earphones of type E_1 and 5 earphones of type E_2 .
Basket V contains 8 batteries of type B_1, 10 batteries of type B_2 and 7 batteries of type B_3.

Part A A customer selects, at random, one item from each basket.

1) Show that the probability of obtaining an earphone E_1 and a battery B_1 is $\frac{6}{25}$.

2) Calculate the probability that an earphone E_1 is among the two selected items.

3) The shop announces the following prices:

Item	Earphone E_1	Earphone E_2	Battery B_1	Battery B_2	Battery B_3
Price in \$	40	15	30	25	50

A customer pays a total amount of **\$65** . Prove that the probability to do that is $\frac{37}{100}$.

Part B In this question, a customer selects, at random, an earphone from basket U and selects simultaneously and at random two batteries from basket V.

Calculate the probability that the customer pays an amount less than or equal to **\$70**

Question V-(20 points)

In order to prevent a certain disease, we vaccinated 40% of persons of a population.

Then we noticed that 85% of the vaccinated persons were not affected by the disease and that 75% of the persons who were not vaccinated are affected by the disease.
A person is chosen randomly from this population. Consider the following events:

D : « the chosen person is affected by the disease».
V : « the chosen person is vaccinated ».

1) a- Verify that the probability of the event $D \cap V$ is equal to $\frac{6}{100}$.

b- What is the probability that the chosen person is affected by the disease and is not vaccinated?

c- Deduce the probability P(D).

2) The chosen person is not affected by the disease.
Calculate the probability that he/she is vaccinated.

3) In this part, suppose that this population is formed of 300 persons.
We choose randomly 3 persons from this population.
What is the probability that at least 1 , among the 3 chosen persons, is vaccinated ?

Question VI-(20 points)

Let f be the function defined over $]1; +\infty[$ by $f(x) = x - \frac{1}{xLnx}$ and designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $\lim_{x \to 1} f(x)$ and deduce an asymptote to (C).

2) Calculate $\lim_{x \to +\infty} f(x)$. Prove that the straight line (L) of equation y = x is an asymptote to (C) . **Then** , study the relative position of (C) and (L).

3) Calculate f '(x) and show that f(x) is strictly increasing.
Then , set-up the table of variations of f(x) .

4) Show that the equation f(x) = 0 has a unique root α and verify that $1.5 < \alpha < 1.6$.

5) Draw (L) and (C).

6) a- Calculate the area A(t) of the region limited by the curve (C) , the straight line (L) and the two straight lines of equations x = e and x = t where $t > e$.

b- Show that for all $t > e$, we have A(t)< t.

Question VII-(20 points)

Part A Let h be the function defined over $]0;+\infty[$ by $(C): h(x)=\dfrac{e^x-1}{x}$.

1) a- Verify that $h'(x)=\dfrac{(x-1)e^x+1}{x^2}$.

b- Let g be the function defined over $]0;+\infty[$ by $g(x)=(x-1)e^x$.

Set-up the table of variations of g(x) and deduce that h'(x) > 0.

2) a- Calculate $\lim\limits_{x\to 0} h(x)$, $\lim\limits_{x\to+\infty} h(x)$ and $\lim\limits_{x\to+\infty} \dfrac{h(x)}{x}$.

b- Set up the table of variations of h(x).

3) a- Write an equation of (Δ), the tangent to (C) at the point with abscissa 1.

b- Draw (Δ) and (C).

Part B Consider the function f(x) defined over $]0;+\infty[$ by $(\Gamma): f(x)=h(x)+Lnx$

1) a- Calculate $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.

b- Set up the table of variations of the function f(x).

2) a- Prove that the equation f(x) = 0 has a unique solution α where $0.3<\alpha<0.4$.

b- Compare $h(\alpha)$ and h(1). Deduce that $Ln\alpha>1-e$.

3) a- Discuss, according to the values of x, the relative positions of (C) and (Γ).

b- Draw (Γ) in the same system of axes with (Δ) and (C).

Question VIII-(20 points)

Consider, in the complex plane, the point M with affix z ($z\neq 0$). The point M' with affix z' is associated with point M such that $z'=\dfrac{2}{\bar{z}}$.

1) Let $z=re^{i\theta}$ (r > 0), write z' in the exponential form.

2) a- Show that $OM\times OM'=2$.

b- If z = z', prove that M moves on a circle (C) whose center and radius are to be determined.

3) Let z = 1 + iy where y is a real number.

a- Prove that $|z'-1|=1$.

b- As y varies, show that M' moves on a circle (C') whose center and radius are to be determined

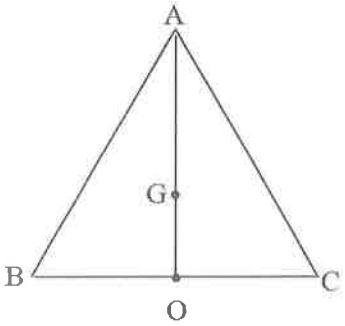
Question IX-(20 points)

Consider a direct equilateral triangle ABC with center G.

O is the midpoint of [BC] and F that of [AC].

Let S be the direct plane similitude that transforms A onto B and G onto C.

Part A



1) a- Determine the ratio and an angle of S.
   b- Prove that F is the center of S.

2) a- Determine the line (L), image of (AB) under S.

   b- Construct the point D, image of B under S.

Part B

The complex plane is a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$ such that:

$z_A = i\sqrt{3}$ and $z_C = 1$.

1) Find the complex form of S.

2) Determine the nature and characteristic elements of $S \circ S$.

3) Express $\overline{FD}$ in terms of $\overline{FA}$.

Question X-(20 points)

Consider the sequence (u_n) defined as $\begin{cases} u_0 = 3 \\ u_{n+1} = \dfrac{u_n - 8}{u_n - 5} \end{cases}$ for $n \in N$.

1) Calculate u_1 and u_2. Deduce that (u_n) is neither arithmetic nor geometric.
2) a- Prove, by mathematical induction, that $2 < u_n \leq 3$.
   b- Show that (u_n) is strictly decreasing.
   c- Deduce that (u_n) is convergent.
3) Consider another sequence (v_n) defined by the relation: $v_n = \dfrac{u_n - 2}{u_n - 4}$.
   a- Prove that (v_n) is geometric. Find also its first term.
   b- Express v_n in terms of n.
   c- Prove that $u_n = \dfrac{2(3)^n + 4}{3^n + 1}$. Then, deduce the limit of u_n.

12M — Mathematics Mock Exam — June, 2025

I

1) $e^{x+1}-1>0,\ e^{x+1}>1,\ x+1>0,\ x>-1$ (b) 4

2) $\lim\limits_{x\to 0^+} x(1+\ln x) = \lim\limits_{x\to 0^+}(x+x\ln x) = 0^+ + 0^- = 0$ (c) 4

3) $\lim\limits_{x\to+\infty}\left[f(x)-(-x-1)\right] = \lim\limits_{x\to+\infty}\dfrac{x+2}{e^{x-1}} = \lim\limits_{x\to+\infty}\dfrac{1}{e^{x-1}} = \dfrac{1}{+\infty} = 0^+$ (c) 4

4) 2R, 5B — 3 balls drawn successively with replacement. At least 1R.

$N = 7^3 - 5^3 = 218$ ways (c) 4

(7^3: all ways; 5^3: all black)

5) A and B independent $P(A\cap B) = P(A)\cdot P(B) = 0.4\,P(A)$ 4

$P(\bar{B}/A) = \dfrac{P(\bar{B}\cap A)}{P(A)} = \dfrac{P(A)-P(A\cap B)}{P(A)} = \dfrac{0.6P(A)}{P(A)} = 0.6$ (c)

II

$A(4,0,1)\quad B(2,1,2)\quad C(2,0,3)\quad E(3,-1,0)$

1) (a) $\overrightarrow{AM}\cdot(\overrightarrow{AB}\times\overrightarrow{AC}) = 0,\ \begin{vmatrix} x-4 & y & z-1 \\ -2 & 1 & 1 \\ -2 & 0 & 2 \end{vmatrix} = 0$ 3

$(x-4)(2) + y(-4) + (z-1)(2) = 0,\ x+y+z-5=0 \ldots (P)$

(b) $A\in(P)$ and $\overrightarrow{EA}(1,1,1) = \vec{n}_{(P)}$ 2 verified

2) (a) $\overrightarrow{BC}(0,-1,1)\cdot\overrightarrow{AB}(-2,1,1) = 0-1+1=0 \to \hat{ABC} = 90°$ 2

(b) $\text{Area}_{ABC} = \frac{1}{2}\sqrt{4+1+1}\cdot\sqrt{0+1+1} = \frac{1}{2}\sqrt{12} = \sqrt{3}\ \text{unit}^2$ 3

(c) $\text{Volume} = \frac{1}{3}EA\cdot\text{Area}_{ABC} = \frac{1}{3}(\sqrt{3})(\sqrt{3}) = 1\ \text{unit}^3$ 3

II 3) $(Q): x-2y-2z-2=0$ $\quad x_A-2y_A-2z_A-2=4-0-2-2=0$ $\Rightarrow A\in(Q)$

$\vec{BE}(1,-2,-2)=\vec{n_Q}(1,-2,-2)$ verified 2

4) (a) $(BC):\begin{cases} x=2 \\ y=-t+1 \\ z=t+2 \end{cases}$ 2

(b) $M\in(BC)$ If the distance from M to (Q) is constant then $(BC)\parallel(Q)$. 3

$x_{BC}-2y_{BC}-2z_{BC}-2=0$, $2+2t-2-2t-4-2=0$

$-6=0$ impossible then $(BC)\nparallel(Q)$ → verified

III $(C): f(x)=(x+1)^2e^{-x}$ $\quad D_f=]-\infty,+\infty[$

1) (a) $\lim\limits_{x\to-\infty} f(x)=\infty\cdot\infty=+\infty$ $\quad f(-2)=e^2\approx 7.4$ 1

(b) $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\dfrac{(x+1)^2}{e^x}=\lim\limits_{x\to+\infty}\dfrac{2(x+1)}{e^x}=\lim\limits_{x\to+\infty}\dfrac{2}{e^x}=0^+$ 1

Then $y=0$ HA 1

2) $f'(x)=2(x+1)e^{-x}-e^{-x}(x+1)^2=(x+1)e^{-x}[2-x-1]$

$f'(x)=(1-x^2)e^{-x}$ 1

x	$-\infty$		-1		1		$+\infty$
f'		$-$	0	$+$	0	$-$	
f	$+\infty$	↘	min	↗	max $4/e$	↘	0^+

3

3) $f(1.4)=1.42>1.4$
$f(1.5)=1.39<1.5$ } verified 1

5) $F(x)=(px^2+qx+r)e^{-x}$

(a) $F'(x)=(2px+q)e^{-x}-e^{-x}(px^2+qx+r)$

$F'(x)=[-px^2+(2p-q)x+q-r]e^{-x}\equiv f(x)=(x^2+2x+1)e^{-x}$

III 5) (a) $x^2: -p = 1 \rightarrow \boxed{p = -1}$ $\quad F(x) = (-x^2 - 4x - 5)e^{-x}$

$x: 2p - q = 2 \rightarrow -2 - q = 2 \rightarrow \boxed{q = -4}$ 4

$q - r = 1 \rightarrow -4 - r = 1 \rightarrow \boxed{r = -5}$

(b) $\text{Area} = \int_0^1 f(x)\,dx = F(x)\Big|_0^1 = F(1) - F(0)$ 3

$\text{Area} = (-10e^{-1}) - (-5) = \left(5 - \frac{10}{e}\right) \text{unit}^2$

IV A

$15E_1$		$8B_1$
$5E_2$		$10B_2$
		$7B_3$
u		v

One item from each basket

1) $P(E_1B_1) = \frac{15}{20} \times \frac{8}{25} = \frac{6}{25}$ 5

2) $P(E_1B_1 \text{ or } E_1B_2 \text{ or } E_1B_3) = \frac{6}{25} + \frac{15}{20} \times \frac{10}{25} + \frac{15}{20} \times \frac{7}{25} = \frac{3}{4}$ 5

3) \$65 $\begin{cases} \$40 + \$25 \rightarrow E_1B_2 \\ \$15 + \$50 \rightarrow E_2B_3 \end{cases}$ 5

$P = \frac{15}{20} \times \frac{10}{25} + \frac{5}{20} \times \frac{7}{25} = \frac{37}{100}$

B $\leq \$70$ $\begin{cases} \underset{E_1}{\$40} + \ldots \text{ No case} \\ \underset{E_2}{\$15} + 2B_2 \text{ or } B_1B_2 \end{cases}$ 5

$P = \frac{5}{20}\left[\frac{{}_{10}C_2}{{}_{25}C_2} + \frac{{}_8C_1 \cdot {}_{10}C_1}{{}_{25}C_2}\right] = \frac{5}{48}$

V

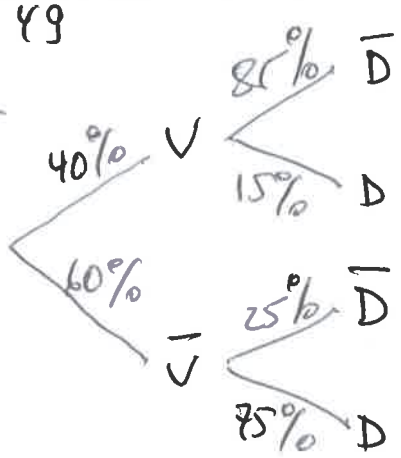
1) (a) $P(D \cap V) = P(V) \times P(D/V) = 0.4(0.15) = 0.06 = \frac{6}{100}$ 4

(b) $P(D \cap \bar{V}) = P(\bar{V}) \times P(D/\bar{V}) = 0.6(0.75) = 0.45 = \frac{45}{100}$ 4

(c) $P(D) = P(D \cap V) + P(D \cap \bar{V}) = \frac{51}{100}$ 4

2) $P(V/\bar{D}) = \frac{P(V \cap \bar{D})}{P(\bar{D})} = \frac{0.4 \times 0.85}{1 - 0.51} = \frac{34}{49} \approx 0.694$ 4

3) $P(A) = 1 - P(\bar{A}) = 1 - \frac{{}^{180}C_3}{{}^{300}C_3} \simeq 0.785$ 4



VI (c): $f(x) = x - \frac{1}{x \ln x}$ $\quad D_f =]1, +\infty[$

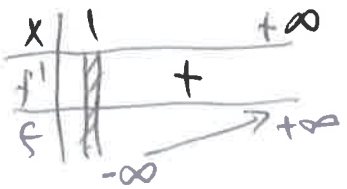
1) $\lim_{x \to 1} f(x) = 1 - \infty = -\infty$ 1 $\quad x = 1$ UA 1

2) $\lim_{x \to +\infty} f(x) = +\infty - 0 = +\infty$ 1 $\quad \lim_{x \to +\infty} [f(x) - x] = \lim_{x \to +\infty} \frac{-1}{x \ln x} = 0^-$ 1 verified

$f(x) - x = \frac{-1}{x \ln x} < 0$ 1 $\quad x > 1$, $\ln x > 0$, $x \ln x > 0$ $\quad$ (c) below (L)

3) $f'(x) = 1 + \frac{1}{(x \ln x)^2}\left[1 \cdot \ln x + \frac{1}{x} \cdot x\right] = 1 + \frac{\ln x + 1}{(x \ln x)^2} > 0$ 1 $\quad \ln x > 0$, $(x \ln x)^2 > 0$

Then $f(x)$ strictly increases.



2

4) f is continuous and strictly increases from $-\infty < 0$ to $+\infty > 1$

Then $f(x) = 0$ has a unique root.

$f(1.5) \times f(1.6) = -0.039 < 0$ 2 verified

VI 6) ⓐ $A(t) = \int_e^t [x - f(x)]\,dx = \int_e^t \frac{1}{x \ln x}\,dx$ 3

$u = \ln x$, $du = \frac{dx}{x}$

$A(t) = \int_1^{\ln t} \frac{du}{u} = \ln|u| \Big|_1^{\ln t} = \ln|\ln t|$ unit^2

ⓑ $|\ln t| < t$, $\ln|\ln t| < \ln t < t \rightarrow A(t) < t$ 2

VII Part A (C): $h(x) = \frac{e^x - 1}{x}$ $x > 0$ 1

1) ⓐ $h'(x) = \frac{e^x \cdot x - (e^x - 1)}{x^2} = \frac{(x-1)e^x + 1}{x^2}$

ⓑ $g(x) = (x-1)e^x$ $x > 0$ $g'(x) = e^x + (x-1)e^x = xe^x > 0$

2

x	0		$+\infty$
g'		$+$	
g	-1	↗	$+\infty$

Then $g(x) > -1$, $g(x) + 1 > 0$ 1

$(x-1)e^x + 1 > 0 \rightarrow h'(x) > 0$

2) ⓐ $\lim_{x \to 0} h(x) = \lim_{x \to 0} \frac{e^x}{1} = 1$ 1 $\lim_{x \to +\infty} h(x) = \lim_{x \to +\infty} \frac{e^x}{1} = +\infty$ 1

$\lim_{x \to +\infty} \frac{h(x)}{x} = \lim_{x \to +\infty} \frac{e^x - 1}{x^2} = \lim_{x \to +\infty} \frac{e^x}{2x} = \lim_{x \to +\infty} \frac{e^x}{2} = +\infty$ 1

ⓑ

x	0		$+\infty$
h'		$+$	
h	1	↗	$+\infty$

2

3) ⓐ $x_A = 1$, $y_A = e - 1$ $h'(1) = 1$ $y - (e-1) = 1(x-1)$

Then (Δ): $y = x + e - 2$ 1

VII part B $(\Gamma): f(x) = h(x) + \ln x = \frac{e^x - 1}{x} + \ln x \quad x > 0$

1) (a) $\lim_{x \to 0} f(x) = 1 - \infty = -\infty$, $\lim_{x \to +\infty} f(x) = +\infty + \infty = +\infty$

(b) $f'(x) = h'(x) + \frac{1}{x} > 0$

x	0		$+\infty$
f'	‖	$+$	
f	‖	↗	$+\infty$
	$-\infty$		

2) (a) f is continuous and strictly increases from $-\infty < 0$ to $+\infty > 0$ then $f(x) = 0$ has a unique root.

$f(0.3) \times f(0.4) = -0.081 < 0$ verified

(b) $0.3 < \alpha < 0.4 \to \alpha < 1$ but $h(x)$ strictly increases then $h(\alpha) < h(1)$ but $f(\alpha) = 0$, $h(\alpha) + \ln \alpha = 0$

$h(\alpha) = -\ln \alpha$

$-\ln \alpha < e - 1 \to \ln \alpha > 1 - e$

3) (a) $h(x) - f(x) = -\ln x$

(c) above (Γ) for $0 < x < 1$

(c) below (Γ) for $x > 1$

(c) cuts (Γ) at $(1, e-1)$

VIII $z_M = z \quad z_{M'} = z' = \frac{2}{\bar{z}}$

1) $z = re^{i\theta} \to z' = \frac{2}{re^{-i\theta}} = \frac{2}{r} e^{i\theta}$ 5

2) (a) $|z'| = \frac{2}{r}$, $|z| = r \quad OM \times OM' = |z| \times |z'| = \frac{2}{r} \times r = 2$

(b) $z = z' \to z = \frac{2}{\bar{z}} \to z\bar{z} = 2 \to |z|^2 = 2 \to |z| = \sqrt{2}$ 5

M moves on the circle $c(0, \sqrt{2})$

VIII 3) $z = 1 + iy \quad y \in \mathbb{R}$ — 5

(a) $|z'-1| = \left|\frac{z}{\bar{z}} - 1\right| = \left|\frac{z-\bar{z}}{\bar{z}}\right| = \frac{|2-1-iy|}{|1-iy|} = \frac{|1-iy|}{|1-iy|} = 1$

(b) $|z'-1| = 1$ Let $z_A = 1 \rightarrow AM' = 1$ — 5

$M' \in$ the circle $C'(A,1)$ where $A(1,0)$

IX A 1) (a) $K_S = \frac{BC}{AG} = \frac{2OC}{\frac{2}{3}AO} = 3\tan 30° = \sqrt{3}$ — 2

$\alpha_S = (\vec{AG}, \vec{BC}) = (\vec{GO}, \vec{OC}) = \frac{\pi}{2}\ (2\pi)$ — 2

(b) $\frac{BE}{AF} = \tan 60° = \sqrt{3} = K_S$, $(\vec{FA}, \vec{FB}) = \frac{\pi}{2} = \alpha_S$ — 3

S

$A \rightarrow B$

$G \rightarrow C$

$F \rightarrow F$

$B \rightarrow D$

2) (a) $[(AB),(L)] = \frac{\pi}{2}$ or $(AB) \perp (L)$ — 4

and $S(A) = B \Rightarrow (L) \perp (AB)$ at B

(b) $(\vec{FB}, \vec{FD}) = \frac{\pi}{2} \rightarrow \{D\} = (\Delta) \cap (AFC)$ — 2

B $z_A = i\sqrt{3} \qquad z_C = 1$

1) $z' = az + b \qquad a = Ke^{i\alpha} = \sqrt{3}e^{i\frac{\pi}{2}} = i\sqrt{3}$

$z_B = i\sqrt{3}\, z_A + b$, $-1 = i\sqrt{3}(i\sqrt{3}) + b = -3 + b$, $b = 2$

$z' = (i\sqrt{3})z + 2$ — 3

2) $\alpha_{SoS} = \pi \qquad |K_{SoS}| = 3 \rightarrow SoS = h = h(F,-3)$

$h(F) = SoS(F) = S(F) = F$ — 2

3) $h(A) = SoS(A) = S(B) = D$

Then $\vec{FD} = -3\vec{FA}$ — 2

h

$F \rightarrow F$

$A \rightarrow D$

X 1) $u_1 = \frac{3-8}{3-5}$, $u_1 = \frac{5}{2}$ 1 $\quad u_2 = \frac{\frac{5}{2}-8}{\frac{5}{2}-5}$, $u_2 = \frac{11}{5}$ 1

$u_1 - u_0 = \frac{1}{2} \neq u_2 - u_1 = \frac{-3}{10}$ (u_n) : not arithmetic

$\frac{u_1}{u_0} = \frac{5}{6} \neq \frac{u_2}{u_1} = \frac{22}{25}$ (u_n) : not geometric 2

2) (a) $u_{n+1} = 1 - \frac{3}{u_n - 5}$ $\quad 2 < u_0 = 3 \leq 3$ true for $n=0$

$u_n - 5 \;\big|\; u_n - 8$ 1

$-3 \;\big|\; u_n - 5$ (−) (+)

Assume $2 < u_K \leq 3$, $-3 < u_K - 5 \leq -2$

$1 < \frac{-3}{u_K - 5} \leq \frac{3}{2} \rightarrow 2 < 1 - \frac{3}{u_K - 5} \leq \frac{5}{2} \leq 3$ 2

Then $2 < u_{K+1} \leq 3$

(b) $u_0 = 3 > u_1 = \frac{5}{2}$ true for $n=0$

Assume $u_K > u_{K+1} \rightarrow u_K - 5 > u_{K+1} - 5$

$\frac{-3}{u_K - 5} > \frac{-3}{u_{K+1} - 5}$, $1 - \frac{3}{u_K - 5} > 1 - \frac{3}{u_{K+1} - 5}$, $u_{K+1} > u_{K+2}$ verified 2

(c) Since (u_n) strictly decreases with 2 as a lower bound then (u_n) converges. 1

3) $v_n = \frac{u_n - 2}{u_n - 4}$

(a) $v_{n+1} = \frac{u_{n+1} - 2}{u_{n+1} - 4} = \frac{\frac{u_n - 8}{u_n - 5} - 2}{\frac{u_n - 8}{u_n - 5} - 4} = \frac{u_n - 8 - 2u_n + 10}{u_n - 8 - 4u_n + 20}$

$v_{n+1} = \frac{2 - u_n}{12 - 3u_n}$ $\quad \frac{v_{n+1}}{v_n} = \frac{\frac{2-u_n}{3(4-u_n)}}{\frac{u_n - 2}{u_n - 4}} = \frac{1}{3} = r$

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X 3) (a) (V_n) is geometric with $r=\frac{1}{3}$ and first term

$$V_0=\frac{U_0-2}{U_0-4}=\frac{3-2}{3-4}=\frac{1}{-1},\ V_0=-1$$ 4

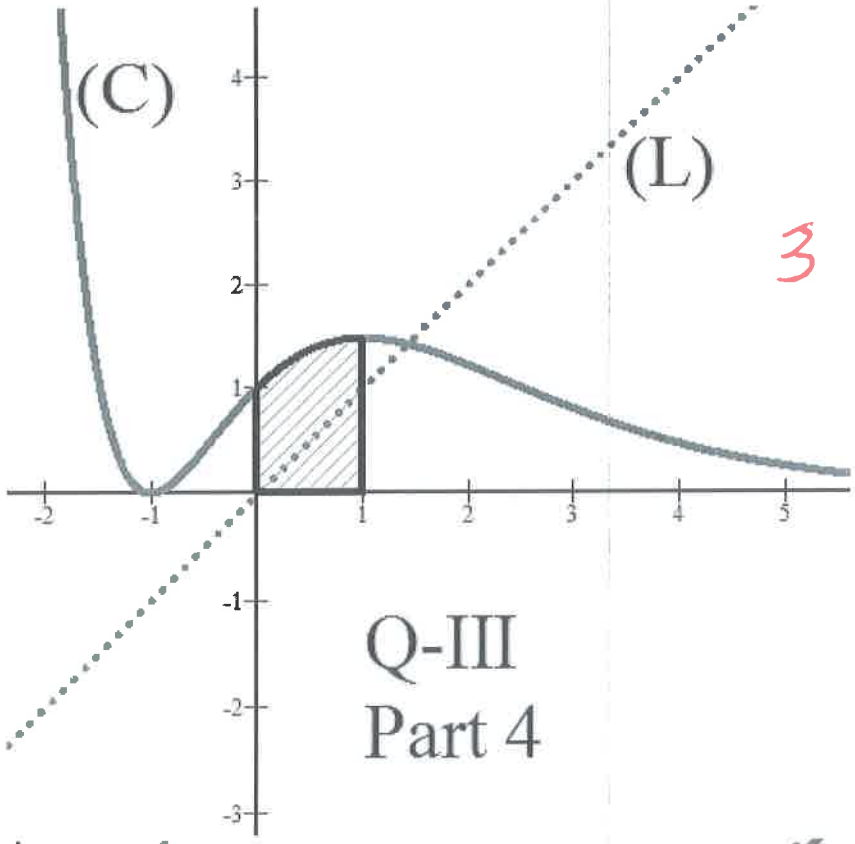
(b) $V_n=V_0\,r^n=(-1)\left(\frac{1}{3}\right)^n \rightarrow V_n=\frac{-1}{3^n}$ 2

(c) $V_n=\frac{U_n-2}{U_n-4}$, $U_nV_n-4V_n=U_n-2$

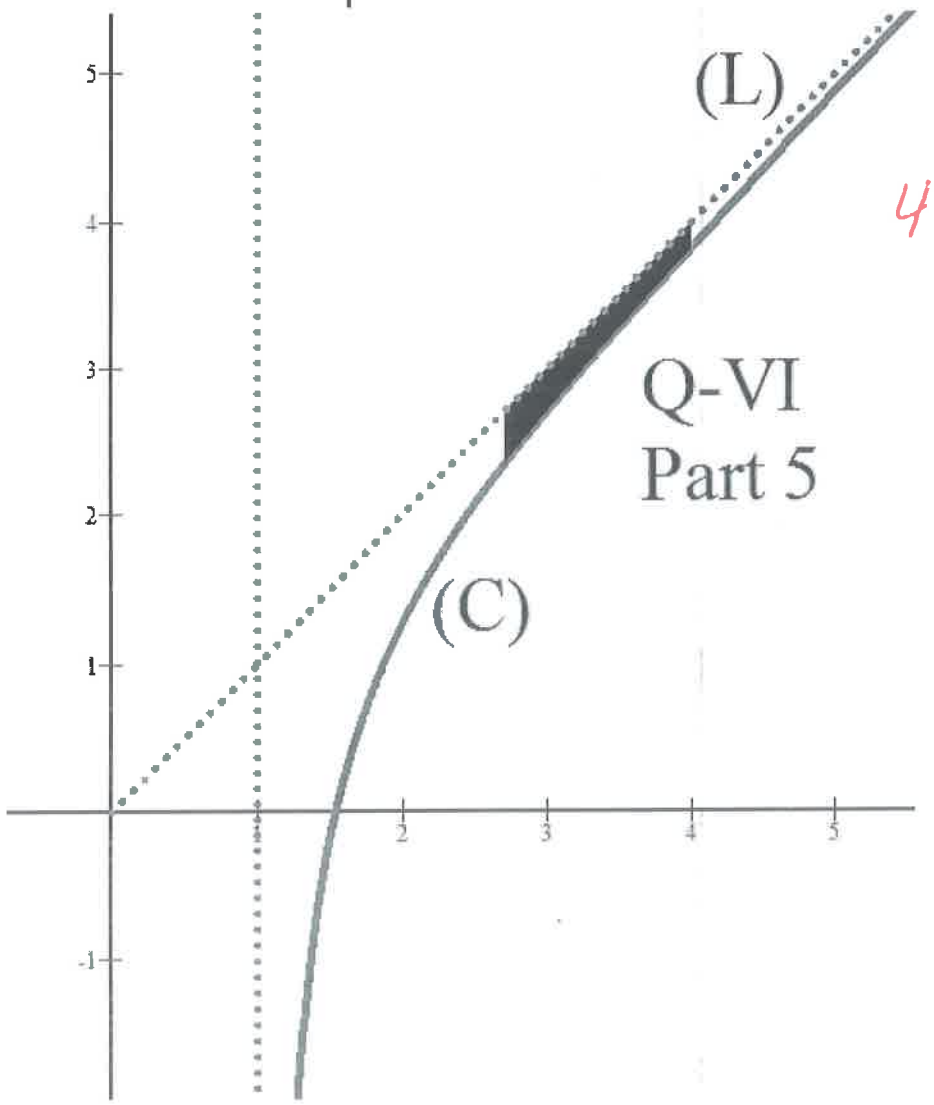
$$U_n=\frac{4V_n-2}{V_n-1}=\frac{\frac{-4}{3^n}-2}{\frac{-1}{3^n}-1}=\frac{\frac{4}{3^n}+2}{\frac{1}{3^n}+1}$$

Then $U_n=\frac{2(3^n)+4}{3^n+1}$ 3

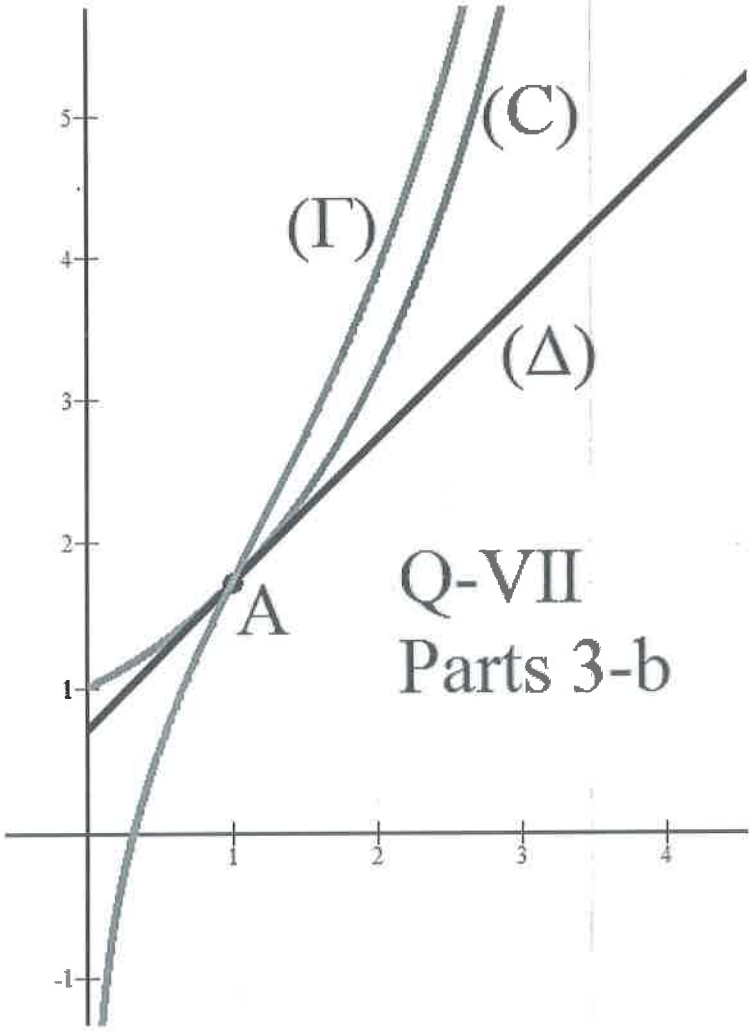
$$\lim_{n\to+\infty} U_n=\lim_{n\to+\infty}\frac{\frac{4}{3^n}+2}{\frac{1}{3^n}+1}=\frac{0+2}{0+1}=2$$ 2



3

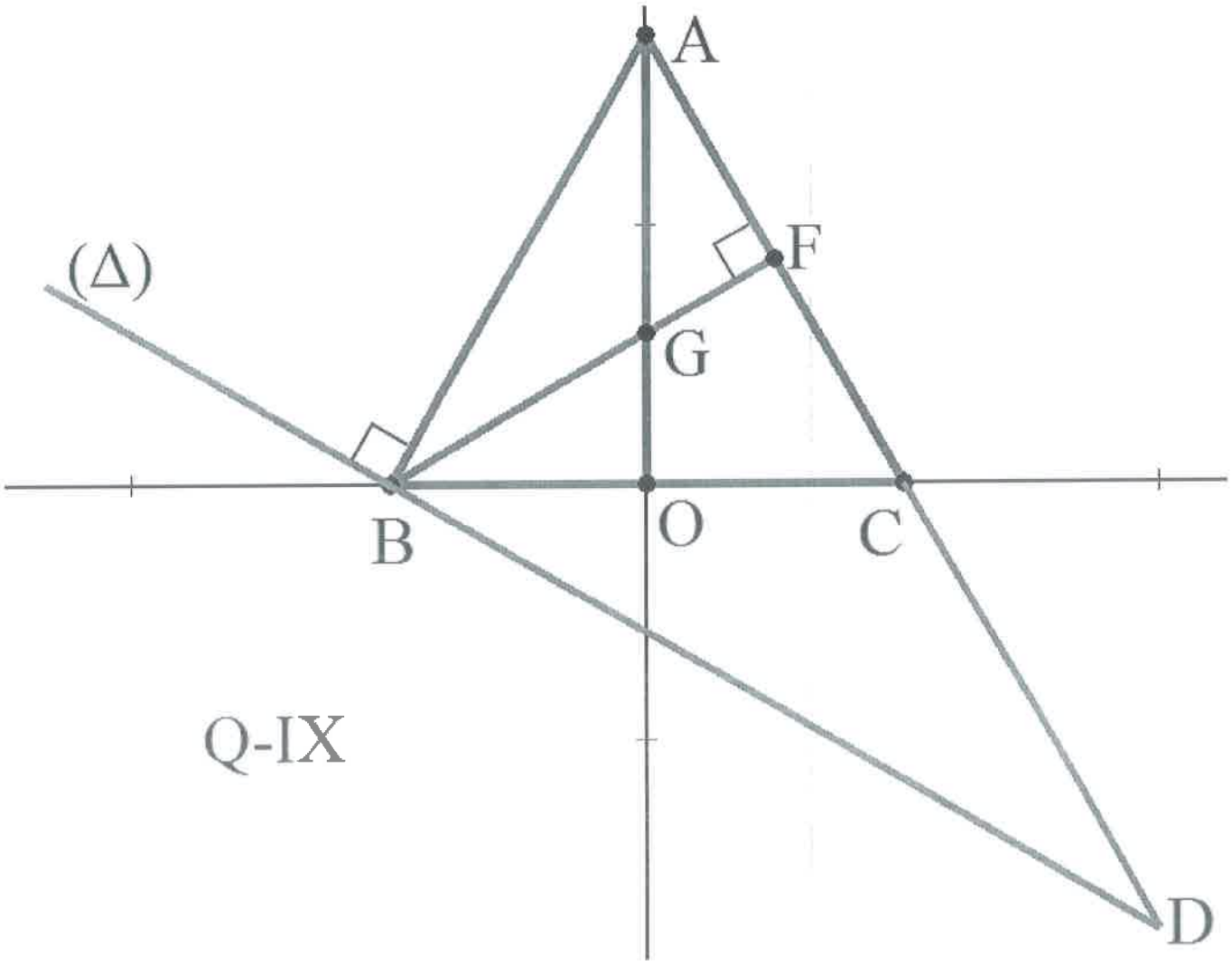


4



Q-VII
Parts 3-b

2



Q-IX