This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School
for Boys and Girls — Loueizeh - Baabda

Stage Exam 01
06NOV2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	11SE	
Teacher :	Mr. George El Haber	
Duration :	1 hour	
Calculators are allowed Don't write your answers on the question sheet paper		
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I. Discuss and solve the following equations where **m** is a real parameter.

1) $mx^2 - (m - 2)x + 2(1 - m) = 0$ $(m \neq 0)$ [5 pts.]

2) $x^2 + 2(m + 3)x - 2m - 7 = 0$ [5 pts.]

II. Solve the following system: [5 pts.]

1) $\begin{cases} x + y = 10 \\ xy = -24 \end{cases}$

III. Consider the expression $E = \frac{x'}{x'' - 2x'} + \frac{x''}{x' - 2x''}$. Without calculating x' and x", calculate the numerical value of E such that x' and x" are roots of the equation: $x^2 - 2x - 1 = 0$. [10 pts.]

IV. Given: $(E) = 2mx^2 + 2(m - 1)x + m - 1 = 0$

1) Determine the set of values of m so that the equation (E) admits two real roots x' and x". $(m \neq 0)$. [5 pts.]

2) a) Find m so that: $x'^2 + x''^2 = 1$. [5 pts.]

b) Prove that $\frac{1}{x'} + \frac{1}{x''} = constant$ [5 pts.]

3) Find between x' and x" when they exist a relation independent of m [5 pts.]

V. Solve if possible the following quadratic equations by completing the square:

1) $5x^2 - 29x + 20 = 0$. [2.5 pts.] 2) $2x^2 + 3x + 5 = 0$. [2.5 pts.]

SEQI / 20232024 / MATHS / IISE Answer Key

I. 1) $mx^2-(m-2)x+2(1-m)=0 \quad (m\neq 0)$

$a-b+c=0 \qquad a=m \quad b=-(m-2) \quad c=2(1-m)$

$m+(m-2)+2(1-m)=0$
$m+m-2+2-2m=0$ (5)
$2m-2+2-2m=0$
$0=0$ ✓

So $x'=-1$
$x''=\frac{-c}{a}=\frac{-(2(1-m))}{m}=\frac{-(2-2m)}{m}=\frac{-2+2m}{m}=\frac{2m-2}{m}$

2) We use Δ

$\Delta=b^2-4ac$
$=(-m+2)^2-4(m)(2-2m)$
$=(-m)^2+2(-m)(2)+2^2-4m(2-2m)$
$=m^2-4m+4-8m+8m^2$
$=9m^2-12m+4 \qquad a=9 \quad b=-12 \quad c=4$

We use Δ again

$\Delta=b^2-4ac$
$=(-12)^2-4(9)(4)$
$=144-144$
$=0$

$\Delta=0 \quad m'=m''=\frac{b}{2a}=\frac{12}{2(9)}=\boxed{\frac{2}{3}}$
double root

2) $x^2 + 2(m+3)x - 2m - 7 = 0$ $\quad a=1 \quad b=2(m+3) \quad c=-2m-7$

$a+b+c=0$
$1 + 2m + 6 + (-2m-7) = 0$
$1 + 2m + 6 - 2m - 7 = 0$ (5)
$1 - 1 = 0$
$0 = 0$

$x' = -1$
$x'' = 2m+7$

or We use Δ

$\Delta = b^2 - 4ac$
$= (2m+6)^2 - 4(-2m-7)$
$= (2m)^2 + 2(2m)(6) + 6^2 - 4(-2m-7)$
$= 4m^2 + 24m + 36 + 8m + 28$
$= 4m^2 + 32m + 64$

We use Δ again $\quad a=4 \quad b=32 \quad c=64$

$\Delta = b^2 - 4ac$
$= (32)^2 - 4(4)(64)$
$= 1,024 - 1,024$
$= 0$

$\Delta = 0 \quad m' = m'' = -\frac{b}{2a} = \frac{-32}{2(4)} = \frac{-32}{8} = \boxed{-4}$
double root

II- 1) $\begin{cases} x+y=10 \\ xy=-24 \end{cases}$ so $S=10$ and $P=-24$

So S and P are the roots of the equation $x^2-Sx+P=0$

So $x^2-10x-24=0$ $\quad a=1 \quad b=-10 \quad c=-24$

$\Delta = b^2-4ac$
$= (-10)^2-4(-24)$
$= 100+96$
$= 196 > 0$ 2 distinct roots

(5)

So $x' = \dfrac{-b+\sqrt{\Delta}}{2a}$ $\quad y' = \dfrac{-b-\sqrt{\Delta}}{2a}$

$= \dfrac{10+\sqrt{196}}{2}$ $\quad = \dfrac{10-\sqrt{196}}{2}$

$= 12$ $\quad = -2$

So $x' = 12$
$y' = -2$

III - $E = \frac{x'}{x''-2x'} + \frac{x''}{x'-2x''}$ $\qquad x^2-2x-1=0$

1) $E = \frac{x'}{x''-2x'} + \frac{x''}{x'-2x''} = \frac{x'(x'-2x'')+x''(x''-2x')}{(x''-2x')(x'-2x'')}$

$= \frac{(x')^2+(x'')^2-4(x')(x'')}{5x'x''-2(x'^2+x''^2)}$

$= \frac{S^2-6P}{9P-2S^2}$ (3)

2) The sum of roots of the equation x^2-2x-1 is:

$S = -\frac{b}{a} = 2$

The product of roots of the equation x^2-2x-1 is:

$P = \frac{c}{a} = -1$ $\qquad$ So $S=2$ and $P=-1$

So $\frac{S^2-6P}{9P-2S^2} = \frac{2^2-6(-1)}{9(-1)-2(2)^2} = \frac{4+6}{-9-8} = \frac{-10}{17}$. (3)

(4)

IV - $(E) = 2mx^2 + 2(m-1)x + m - 1 = 0$

$a = 2m \quad b = 2(m-1) \quad c = m-1$

1) $\Delta = b^2 - 4ac$

$= (2m-2)^2 - 4(2m)(m-1)$

$= (2m)^2 - 2(2m)(2) + (-2)^2 - 4(2m)(m-1)$

$= 4m^2 - 8m + 4 - 8m^2 + 8m - 8m(m-1)$

$= 4m^2 - 8m^2 - 8m + 8m + 4$

$= -4m^2 + 4$

$= 4(-m^2 + 1) \longrightarrow (-m+1)(m+1)$

(5)

$-m+1 = 0$

$-m = -1$

$\boxed{m = 1}$

$m+1 = 0$

$\boxed{m = -1}$

x	$-\infty$		-1		$+\infty$
Δ		$-$	0	$+$ 0	$-$

$m \in]-1; 0[\cup]0, 1[$ or $]-1; 1[$

because 0 is a restriction

2) a) $x'^2 + x''^2 = 1$

$S \qquad P$

$\underline{(x' + x'')^2} = x'^2 + \underline{2(x')(x'')} + x''^2$

$S^2 = x'^2 + 2P + x''^2$

$S^2 - 2P = x'^2 + x''^2$

1 Given

(5)

$S^2 - 2P = 1$

$\left(\frac{2(m-1)}{2m}\right)^2 - 2\left(\frac{m-1}{2m}\right) = 1$

(5)

$$\frac{(m-1)^2}{m^2} - \frac{m-1}{m} = 1$$

$$\frac{m^2-2m+1}{m^2} - \frac{m-1}{m \times m}^{\times m} = 1$$

$$\frac{m^2-2m+1}{m^2} - \frac{m^2-m}{m^2} = 1$$

$$\frac{m^2-2m+1-m^2+m}{m^2} \neq 1$$ Cross multiplication

$$-m+1 = m^2$$

$$-m^2-m+1=0 \qquad a=-1 \quad b=-1 \quad c=1$$

$$\Delta = b^2-4ac$$
$$= (-1)^2-4(-1)(1)$$
$$= 1+4$$
$$= 5 > 0$$ 2 distinct roots

$$m' = \frac{-b+\sqrt{\Delta}}{2a} \qquad m'' = \frac{-b-\sqrt{\Delta}}{2a}$$

$$= \frac{1+\sqrt{5}}{-2} \qquad = \frac{1-\sqrt{5}}{-2}$$

$$\simeq -1.61 \text{ rej.} \qquad \simeq 0.61 \text{ acc.}$$

$$\rightarrow m \in]-1, 0[\cup]0; 1[\leftarrow$$

6

b) $\frac{1}{x'} + \frac{1}{x''} = \text{constant}$ (3)

$(E): 2mx^2 + 2(m-1)x + m - 1 = 0$

$\frac{1}{x'} + \frac{1}{x''} = \frac{x'' + x'}{x'' x'} = \frac{S}{P}$

$S = -\frac{b}{a}$ $\quad P = \frac{c}{a}$

$= \frac{\frac{-2(m-1)}{2m}}{\frac{(m-1)}{2m}}$

$= -2$

3) $S = \frac{-2(m-1)}{2m}$ $\quad P = \frac{m-1}{2m}$

$S = \frac{-2m+4}{2m}$

$mS = -m + 1$

$mS + m = 1$

$m(S+1) = 1$

$m = \frac{1}{S+1}$

$2mP = m - 1$

$2mP - m = -1$

$m(2P - 1) = -1$

$m = \frac{1}{1-2P}$ (5)

$m = m$

$\frac{1}{S+1} = \frac{1}{1-2P}$

$1 - 2P = S + 1$

$S + 2P = 0$

Relation between x' and x''

$S + 2P = 0$

$x' + x'' + 2(x'x'') = 0$

(7)

V. 1) $5x^2 - 29x + 20 = 0$

$$\frac{5x^2}{5} - \frac{29}{5}x + \frac{20}{5} = 0$$

$$x^2 - \frac{29}{5}x + 4 = 0$$ (2.5)

$$x^2 - \frac{29}{5}x + \left(-\frac{29}{10}\right)^2 = -4 + \left(-\frac{29}{10}\right)^2$$

$$x^2 - \frac{29}{5}x + \frac{841}{100} = -4 + \frac{841}{100}$$

$$\sqrt{\left(x - \frac{29}{10}\right)^2} = \pm\sqrt{\frac{441}{100}}$$

$$x - \frac{29}{10} = \pm\frac{21}{10}$$

$$x = \frac{29}{10} \pm \frac{21}{10}$$

$$x = \frac{29 \pm 21}{10} \quad \text{so} \quad x' = \frac{29+21}{10} \qquad x'' = \frac{29-21}{10}$$

$$\boxed{x' = \frac{50}{10}} \qquad \boxed{x'' = \frac{8}{10}}$$

2) $2x^2+3x+5=0$

$\frac{2}{2}x^2+\frac{3}{2}x+\frac{5}{2}=0$

$x^2+\frac{3}{2}x+\frac{5}{2}=0$ (2.5)

$x^2+\frac{3}{2}x+\frac{9}{16}=-\frac{5}{2}+\frac{9}{16}$

$\sqrt{\left(x+\frac{3}{4}\right)^2}=\sqrt{-\frac{31}{16}}$ Impossible