This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 02

26MAR2025

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	11SE
Teacher :	Mr. George Mouhayar
Duration :	1 hour
You are to answer the questions to this exam on : Squared paper	

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I. Given the function $f(x) = \frac{x^2-4x+4}{x+1}$ and (C) its representative curve in an orthonormal system $(O; \vec{\imath}; \vec{\jmath})$ (35 points)

a. Write the domain of definition of $f(x)$ in interval form.

b. Find the limits of $f(x)$ at the open bounds of its domain and deduce any asymptotes.

c. Show that $f(x)$ can be written in the form $f(x) = x - 5 + \frac{9}{x+1}$.

d. Show that the line (d) of equation $y = x - 5$ is an oblique asymptote to (C).

e. Study the relative positions of the curve (C) and the asymptote (d).

f. Calculate $f'(x)$ and show that $f'(x) = \frac{x^2+2x-8}{(x+1)^2}$

g. Set the table of variations of $f(x)$.

h. Determine the coordinates of the x-intercept and the y-intercept.

i. Calculate $f(1), f(-2), and\ f(5)$.

j. On a coordinate system, draw (C).

k. Show that the point I (-1; -6) is a center of symmetry for (C) the representative curve of $f(x)$.

l. Determine the equation of the line tangent to (C) at the point A (1; 0.5).

II. Given the function $f(x) = \frac{3x-4}{x-1}$ (15 points)

a. Write the domain of definition of $f(x)$ in interval form.

b. Find the limits of $f(x)$ at the open bounds of the domain and deduce asymptotes.

c. Calculate $f'(x)$

d. Set up the table of variation of $f(x)$.

e. Show that the point S (1; 3) is a center of symmetry of the graph of $f(x)$.

Solution for SE02 — Class 1

11SE Maths 20242025

I $f(x) = \dfrac{x^2-4x+4}{x+1}$

a) $x+1 \neq 0$

$x \neq -1$

Domain $D_f =]-\infty; -1[\cup]-1; +\infty[$ (1)

b) $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{x^2-4x+4}{x+1} = \lim\limits_{x\to-\infty} \dfrac{x^2}{x} = \lim\limits_{x\to-\infty} x = -\infty$ (1)

$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \dfrac{x^2-4x+4}{x+1} = \lim\limits_{x\to+\infty} \dfrac{x^2}{x} = \lim\limits_{x\to+\infty} x = +\infty$ (1)

$\lim\limits_{x\to-1^-} \dfrac{x^2-4x+4}{x+1} = \dfrac{(-1)^2-4(-1)+4}{-1^-+1} = \dfrac{9}{0^-} = -\infty$ (1)

$\lim\limits_{x\to-1^+} \dfrac{x^2-4x+4}{x+1} = \dfrac{9}{-1^++1} = \dfrac{9}{0^+} = +\infty$ (1)

$x = -1$ is a V.A. (1)

c)

$$\begin{array}{r|l} x+1 & x^2-4x+4 \\ \hline -(x^2+x) & x-5 \\ \hline -5x+4 & \\ -(-5x-5) & \\ \hline 9 & \end{array}$$

(2)

$\Rightarrow f(x) = x-5+\dfrac{9}{x+1}$

d) $\lim_{x \to -\infty} (f(x) - y) = \lim_{x \to -\infty} \left(x - 5 + \frac{9}{x+1} - (x-5)\right)$

$= \lim_{x \to -\infty} \frac{9}{x+1} = 0^-$ (1)

$\lim_{x \to +\infty} (f(x) - y) = \lim_{x \to +\infty} \left(x - 5 + \frac{9}{x+1} - (x-5)\right)$

$= \lim_{x \to +\infty} \frac{9}{x+1} = 0^+$ (1)

Therefore $y = x - 5$ is an oblique asymptote

e) $\lim_{x \to -\infty} (f(x) - y) = 0^- \Rightarrow$ The curve is below the asymptote (1)

$\lim_{x \to +\infty} (f(x) - y) = 0^+ \Rightarrow$ the curve is above the asymptote (1)

f) $f(x) = \frac{x^2 - 4x + 4}{x+1}$ over the whole domain

$f'(x) = \frac{(2x-4)(x+1) - (x^2-4x+4)(1)}{(x+1)^2}$ (1)

$f'(x) = \frac{2x^2 + 2x - 4x - 4 - x^2 + 4x - 4}{(x+1)^2}$ (1)

$f'(x) = \frac{x^2 + 2x - 8}{(x+1)^2}$ (1)

g) $f'(x) = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4 \quad x = 2$

$\Delta > 0 \quad a > 0$ same sign as "a" outside the roots. opposite of "a" between the roots.

$f'(x) > 0$ for $x < -4 \quad x > 2$

$f'(x) < 0$ For $x \in]-4; 2[$

$x \neq -1$

x	$-\infty$		-4		-1		2		$+\infty$
$f'(x)$		$+$	0	$-$	‖	$-$	0	$+$	
$f(x)$	$-\infty$	↗	-12	↘	$+\infty$ ‖ $-\infty$	↘	0	↗	$+\infty$

0.25 for each check mark

$(-4;-12)$ local max (0.5)

$(2;0)$ local min (0.5)

h) x-intercept $\Rightarrow f(x) = 0$

$x^2 - 4x + 4 = 0$

$x = 2$ (0.5)

$(2;0)$

y-intercept $\Rightarrow x = 0$

$f(0) = 4$

$(0;4)$ (0.5)

i) $f(1) = 0.5$ (0.5)

$f(-2) = -16$ (0.5)

$f(5) = 1.5$ (0.5)

j) on the graph paper

k) $f(2a-x)+f(x) \stackrel{?}{=} 2b$ (1) $I(-1;-6)$

$f(-2-x)+f(x) \stackrel{?}{=} 2(-6)$

$$\frac{(-2-x)^2-4(-2-x)+4}{-2-x+1}+\frac{x^2-4x+4}{x+1} \stackrel{?}{=} -12$$

$$\frac{x^2+4x+4+8+4x+4}{-x-1}+\frac{x^2-4x+4}{x+1} \stackrel{?}{=} -12$$

$$\frac{x^2+8x+16}{-(x+1)}+\frac{x^2-4x+4}{x+1} \stackrel{?}{=} -12$$

$$\frac{-x^2-8x-16}{x+1}+\frac{x^2-4x+4}{x+1} \stackrel{?}{=} -12 \quad (4)$$

$$\frac{\cancel{-x^2}-8x-16+\cancel{x^2}-4x+4}{x+1} \stackrel{?}{=} -12$$

$$\frac{-12x-12}{x+1} \stackrel{?}{=} -12$$

$$\frac{-12(\cancel{x+1})}{\cancel{x+1}} \stackrel{?}{=} -12$$

$-12 = -12$

True

l) $A(1;0.5)$

$$m = f'(1) = \frac{1^2+2-8}{(1+1)^2} = -\frac{5}{4}$$

$y-y_A = m(x-x_A)$

$y-0.5 = -\frac{5}{4}(x-1)$ (2)

$y = -\frac{5}{4}x+\frac{5}{4}+\frac{1}{2}$

$y = -\frac{5}{4}x+\frac{7}{4}$

II. $f(x) = \dfrac{3x-4}{x-1}$

a. $x - 1 \neq 0$

$x \neq 1$

Domain : $x \in]-\infty ; 1[\cup]1 ; +\infty[$ (1)

b. $\lim\limits_{x \to -\infty} \dfrac{3x-4}{x-1} = \lim\limits_{x \to -\infty} \dfrac{3x}{x} = 3$ (1)

$\lim\limits_{x \to +\infty} \dfrac{3x-4}{x-1} = \lim\limits_{x \to +\infty} \dfrac{3x}{x} = 3$ (1)

$y = 3$ is a H.A. (0.5)

$\lim\limits_{x \to 1^-} \dfrac{3x-4}{x-1} = \dfrac{-1}{0^-} = +\infty$ (1)

$\lim\limits_{x \to 1^+} \dfrac{3x-4}{x-1} = \dfrac{-1}{0^+} = -\infty$ (1)

$x = 1$ is a V.A. (0.5)

c. $f'(x) = \dfrac{3(x-1) - (3x-4)(1)}{(x-1)^2}$

$= \dfrac{3x - 3 - 3x + 4}{(x-1)^2}$ (2)

$f'(x) = \dfrac{1}{(x-1)^2}$

d. $f'(x) \neq 0$

$f'(x) > 0$ For all $x \in D_f$

x	$-\infty$		$+\infty$
$f'(x)$	$+$	$+$	
$f(x)$	$3 \nearrow +\infty$	$-\infty \nearrow 3$	

3.5

e. $S(1;3)$

$f(2a-x)+f(x) \stackrel{?}{=} 2b$

$f(2-x)+\frac{3x-4}{x-1} \stackrel{?}{=} 6$

$\frac{3(2-x)-4}{2-x-1}+\frac{3x-4}{x-1} \stackrel{?}{=} 6$

$\frac{6-3x-4}{-x+1}+\frac{3x-4}{x-1} \stackrel{?}{=} 6$ 3.5

$\frac{-3x+2}{-(x-1)}+\frac{3x-4}{x-1} \stackrel{?}{=} 6$

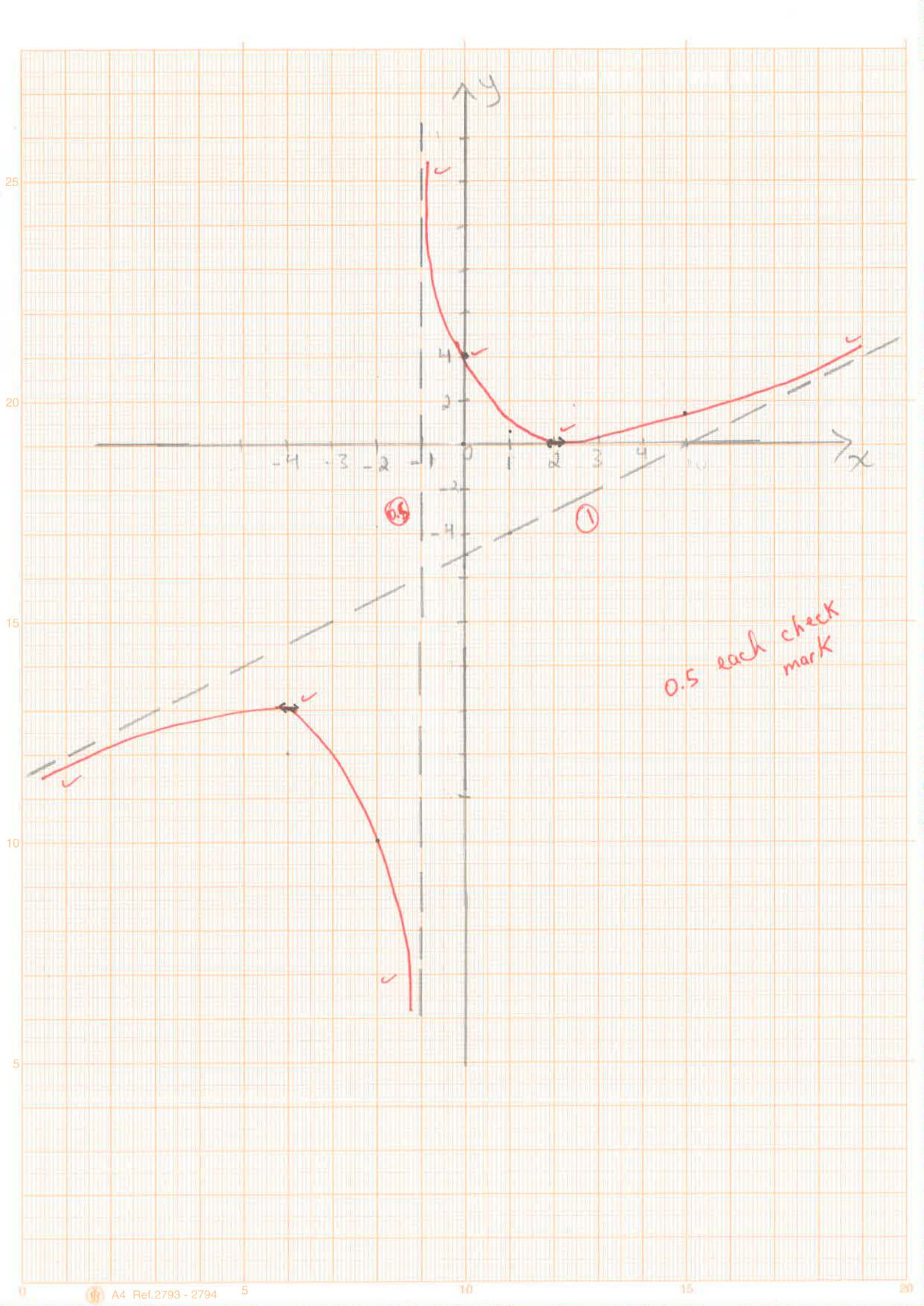
$\frac{3x-2}{x-1}+\frac{3x-4}{x-1} \stackrel{?}{=} 6$

$\frac{6x-6}{x-1} \stackrel{?}{=} 6$

$\frac{6(x-1)}{x-1} \stackrel{?}{=} 6$

$6 = 6$

True


y
x
-4
-3
-2
-1
0
1
2
3
4
4
2
-2
-4
0.5 each check mark