This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Loueizeh - Baabda

Mock Exams : Pt2

13, 14, 15 JUN2022

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	12M
Teacher :	Bishara Kordahi
Duration :	3 hours

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(25 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B with affixes 1 and –2 respectively.

M and M' are two points with respective affixes z and z' such that $z' = \frac{\bar{z}+2}{\bar{z}}$ with $z \neq 0$.

1) Write z in exponential form in the case where $z' = 1+i$.

2) a- Show that $OM' = \frac{BM}{OM}$.

b- If $|z'| = 1$, show that M is on a straight line to be determined.

3) a- For all $z \neq 0$, show that $\bar{z}(z'-1) = 2$.

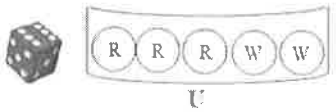
b- For all $z \neq 0$, verify that $\arg(z'-1) = \arg(z) + 2k\pi$ with $k \in Z$.

c- In this part, suppose that $z' = 1+e^{i\theta}$ with $\theta \in]-\pi,\pi]$.

Show that $\overrightarrow{OM} = 2\overrightarrow{AM'}$.

Question II-(25 points)

With a fair die and an urn U containing 3 red balls and 2 white balls the students of grade 12 organized the following game:



U

A player throws the fair die :

If the die shows 1, 2.or 3,the player draws from the urn U **two** balls successively and **without** replacement.

If the die shows 4 or 5, the player draws from the urn U **three** balls successively and **with** replacement.

If the die shows the number 6, the player draws **two** balls from urn U **simultaneously**.

Let ***A*** be the event: the die shows the numbers 1 or 2, or 3. , ***B*** is the event: the die shows numbers 4 or 5

C is the event: the die shows the number 6, ***R*** is the event: the drawn balls are red.

W is the event: the drawn balls are white and ***D*** is the event: the drawn balls have different colors.

1) Calculate P(A) , P(B),and P(C).

2) Calculate P(W/A) and show that $P(W \cap A) = \frac{1}{20}$.

3) Calculate P (W∩B) and deduce that $P(W) = \frac{11}{125}$.

4) Calculate P(R) and deduce that $P(D) = \frac{16}{25}$.

5) a- Show that the probability of drawing one white and one red is $\frac{2}{5}$.

b- The chosen balls are one white and one red. find the probability that the two balls were drawn simultaneously from the urn U.

Question III-(40 points)

In the following table, only one among the proposed answers to each question is correct.
Write down the number of each question and give, **with justification,** the answer which corresponds to it.

N°	Questions	Answers		
		a	b	c
1	If $z=(2-i)^n+(2+i)^n$ where $n\in N$ then z is	pure imaginary	non-zero real number	0
2	For $z\in C$ and $\lvert z\rvert=\sqrt{2}$ then $\left\lvert\frac{\overline{iz+z}}{1+\sqrt{3}}\right\rvert=$	$\sqrt{2}$	$\sqrt{3}-1$	1
3	If $z_A=i$, $z_B=-1$ and $z_M=z$ where $\frac{z-i}{z+1}<0$ (negative real quantity) then M moves on :	(AB)	[AB] without points A and B	(AB) without [AB]
4	If $z=2e^{i\alpha}$ then : $\arg\left(\frac{4-i}{\bar{z}}-z\right)$	$\alpha-\frac{\pi}{2}$	$\frac{\pi}{4}-\alpha$	$2\alpha+\frac{\pi}{2}$
5	If f is the function defined over R by $f(x)=xe^x$, the n^{th} derivative of f(x) =	$(x+n+1)e^x$	$(x+n)e^x$	$(x-n)e^x$
6	$\lim\limits_{x\to+\infty}\left(Ln\left(2e^x+1\right)-x\right)$	2	Ln 2	e^2
7	i^i is equal to	the real number $\frac{1}{\sqrt{e^{\pi}}}$	a pure imaginary number	an indeterminate number
8	The chess game board is an 8 by 8 square grid . The number of rectangles is :	$({}_7C_2)^2=441$	$({}_8C_2)^2=784$	$({}_9C_2)^2=1296$

Question IV-(40 points)

Let f be the function defined over $I =]0;+\infty[$ by $f(x) = x^2 + Lnx$ and (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

Part A :

1) Calculate f '(x) and determine the sense of variations of f over $I =]0;+\infty[$.

2) a- Calculate $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).

b- Calculate $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty} \frac{f(x)}{x}$.

c- Set up the table of variations of f.

d- Deduce that the equation $x^2 + Lnx = 0$ has a unique solution α where $0.6 < \alpha < 0.7$. Study the sign of f(x) according to the values of x

3) a- Prove that (C) has a point of inflection whose abscissa is to be determined.

b- Draw (C).

Part B :

Let (T) be the representative curve of the function h defined over I =]0;+∞[by h(x) = lnx.

1) Study the relative positions of (C) and (T) and draw (T) in the same system as that of (C) .

2) Let g be the function defined over I by $g(x) = x^2 + (Lnx)^2$.

a- Calculate g'(x) and verify that $g'(x) = \frac{2}{x} f(x)$.

b- Deduce the sense of variations of g over I .

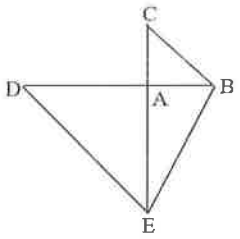
3) Let M_0 be the point of (T) of abscissa α and M any point of (T) of abscissa x.

a- Calculate OM_0^2 in terms of α and OM^2 in terms of x.

b- Prove that $OM_0 \leq OM$ for all x in I.

c- Prove that the tangent at M_0 to (T) is perpendicular to (OM_0).

Question V-(30 points)



In the adjacent figure , ABC and ADE are right isosceles triangles such that AB = 1 , AD = 2 and $(\overrightarrow{AB};\overrightarrow{AC})=(\overrightarrow{AD};\overrightarrow{AE})=\frac{\pi}{2}$ [2π].
O is the midpoint of [BE] .

1) Let r be the rotation with center A and angle $\frac{\pi}{2}$, and h be the dilation with center E and ratio 2. Let f = r o h .

   a- Prove that f is a similitude whose ratio and angle are to be determined.

   b- Determine f(O) and f(A) and deduce that (AO) is perpendicular to(CD).

2) Let s be the direct plane similitude that transforms A onto C and D onto A.

   a- Precise the ratio and an angle of s.

   b- Determine the image by s of the straight line (CD) and that of the straight line (AO).

   c- Deduce the center I of s.

3) Let g = f o s and A' be the symmetric of A with respect to D.

   a- Prove that g is the symmetry with center D.

   b- Deduce that f(C) = A'.

4) The complex plane is referred to a direct orthonormal system $(A;\overrightarrow{AB};\overrightarrow{AC})$.

   a- Determine the affixes of the points O, C and A'.

   b- Write the complex form of f and determine the affix of its center Ω.

12M Mock Part2 Mathematics June, 2022

(I) $z_A = 1$ $z_B = -2$ $z_M = z$ $z_{M'} = z'$ $z' = \frac{\bar{z}+2}{\bar{z}}$ $z \neq 0$ 4

1) $z' = 1+i \rightarrow 1+i = 1 + \frac{2}{\bar{z}} \rightarrow \bar{z} = \frac{2}{i} = -2i \rightarrow z = 2i = 2e^{i\frac{\pi}{2}}$ 4

2) (a) $|z'| = \frac{|\bar{z}+2|}{|\bar{z}|} = \frac{|\overline{z+2}|}{|\bar{z}|} = \frac{|z+2|}{|z|} \rightarrow OM' = \frac{|z_M - z_B|}{OM} = \frac{BM}{OM}$ 4 (*)

(b) $|z'| = 1 \rightarrow OM' = 1 \rightarrow BM = OM \rightarrow M \in \perp$ bisector of $[OB]$: $x = -1$ 4

3) (a) $\bar{z}(z'-1) = z'\bar{z} - \bar{z} = \cancel{z} + 2 - \cancel{z} = 2$ verified (*) 4

(b) $\arg(\bar{z}) + \arg(z'-1) = \arg(2)$, $\arg(z'-1) = 0 - \arg(\bar{z})$

$\arg(z'-1) = -[-\arg(z)] = \arg(z)$ (2π) (*) 4

(c) $z' = 1 + e^{i\theta} \rightarrow z' - 1 = e^{i\theta}$ $\bar{z} = \frac{2}{z'-1} = \frac{2}{e^{i\theta}} = 2e^{-i\theta}$

$\frac{\overrightarrow{OM}}{\overrightarrow{AM'}} \overset{?}{=} 2$ $\frac{z_M - z_O}{z_{M'} - z_A} = \frac{z-0}{z'-1} = \frac{z}{z'-1} = \frac{2e^{i\theta}}{e^{i\theta}} = 2$

$\frac{z_{\overrightarrow{OM}}}{z_{\overrightarrow{AM'}}} = 2 \Rightarrow \overrightarrow{OM} = 2\overrightarrow{AM'}$ verified (*) 5

(II) die

3R
2W

Urn U

1) $P(A) = \frac{3}{6} = \frac{1}{2}$ 2 $P(B) = \frac{2}{6} = \frac{1}{3}$ 2

$P(C) = \frac{1}{6}$ 2

2) $P(W/A) = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{10}$ 2 $P(W \cap A) = P(W/A) \cdot P(A) = \frac{1}{20}$ 2

3) $P(W \cap B) = P(W/B) \cdot P(B) = \left(\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}\right)\left(\frac{1}{3}\right) = \frac{8}{375}$ 3

$P(W) = P(W \cap C) + \frac{1}{20} + \frac{8}{375} = \frac{1}{10} \cdot \frac{1}{6} + \frac{1}{20} + \frac{8}{375} = \frac{11}{125}$ 4

4) $P(R) = P(R/A)\cdot P(A) + P(R/B)\cdot P(B) + P(R/C)\cdot P(C)$

$= \frac{{}^3C_2}{{}^5C_2}\cdot\frac{1}{2} + \left(\frac{3}{5}\right)^3\left(\frac{1}{3}\right) + \frac{{}^3C_2}{{}^5C_2}\cdot\frac{1}{6} = \boxed{\frac{34}{125}}$ 3

Now, $P(W) + P(R) + P(D) = 1 \rightarrow P(D) = \boxed{\frac{16}{25}}$ 1

5) (a) $P(1W1R) = \frac{{}^3C_1\,{}^2C_1}{{}^5C_2}\cdot P(A) + \frac{{}^3C_1\,{}^2C_1}{{}^5C_2}\cdot P(C) = \boxed{\frac{2}{5}}$ 2

(b) $P(C/1W1R) = \frac{\frac{{}^3C_1\cdot{}^2C_1}{{}^5C_2}\cdot P(C)}{2/5} = \boxed{\frac{1}{4}}$ 2

III 1) $z = (2-i)^n + (2+i)^n = (\sqrt{5}e^{-i\alpha})^n + (\sqrt{5}e^{i\alpha})^n \quad \alpha = \tan^{-1}1/2$

$= (\sqrt{5})^n e^{-i\alpha n} + (\sqrt{5})^n e^{i\alpha n} = (\sqrt{5})^n\left[\cos\alpha n - i\sin\alpha n + \cos\alpha n + i\sin\alpha n\right]$

$= \boxed{2(\sqrt{5})^n\cos(\alpha n)}$ 5 real $\neq 0$ (b)

2) $|z| = \sqrt{2}$ $\left|\frac{iz+z}{1+\sqrt{3}}\right| = \frac{|iz+z|}{1+\sqrt{3}} = \frac{|z||1+i|}{1+\sqrt{3}} = \frac{\sqrt{2}\cdot\sqrt{2}}{1+\sqrt{3}}$

$= \frac{2}{\sqrt{3}+1} = \boxed{\sqrt{3}-1}$ 5 (b)

3) $z_A = i \quad z_B = -1 \quad z_M = z$ $\quad \frac{z-i}{z+1} < 0,\ \frac{z_M - z_A}{z_M - z_B} < 0,\ \frac{z_{\overrightarrow{AM}}}{z_{\overrightarrow{BM}}} < 0$

M lies between A and B with $M \neq A$ and $M \neq B$

$M \in [AB]$ without A and B (b) 5

4) $z = 2e^{i\alpha}$ $\arg\left(\frac{4-i}{\bar{z}} - z\right) = \arg\left(\frac{4-i-z\bar{z}}{\bar{z}}\right) = \arg\left(\frac{4-i-|z|^2}{\bar{z}}\right)$

$= \arg\left(\frac{4-i-4}{\bar{z}}\right) = \arg(-i) - \arg(\bar{z}) = -\frac{\pi}{2} - (-\alpha) = \alpha - \frac{\pi}{2}$ (a) 5

5) $f(x) = xe^x$, $f'(x) = e^x + xe^x = (x+1)e^x$

$f''(x) = e^x + (x+1)e^x = (x+2)e^x$

$f^{(n)}(x) = (x+n)e^x$ (b) 5

6) $\lim_{x\to+\infty}\left[\ln(2e^x+1)-x\right] = \lim_{x\to+\infty}\left[\ln(2e^x+1)-\ln e^x\right]$

$= \lim_{x\to+\infty}\left[\ln\left(\frac{2e^x+1}{e^x}\right)\right] = \lim_{x\to+\infty} \ln\left(2+\frac{1}{e^x}\right)^{\to 0} = \ln 2$ (b) 5

7) $i^i = \left(e^{i\frac{\pi}{2}}\right)^i = e^{i^2\pi/2} = e^{-\pi/2} = \frac{1}{e^{\pi/2}} = \frac{1}{\sqrt{e^\pi}}$ (a) 5

8) $N_{\text{rectangles}} = {}_9C_2 \cdot {}_9C_2 = \left({}_9C_2\right)^2 = 1296$ different rectangles (c) 5

IV (C): $f(x) = x^2 + \ln x$ $\quad D_f = I =]0, +\infty[$

A i) $f'(x) = 2x + \frac{1}{x} > 0$ (2) $\quad f(x)$ strictly increases (2)

2) (a) $\lim_{x\to 0^+} f(x) = 0 - \infty = -\infty$ (2) $\quad x = 0$ VA (2)

(b) $\lim_{x\to+\infty} f(x) = +\infty + \infty = +\infty$ (2), $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty}\left(x + \frac{\ln x}{x}^{\to 0}\right) = +\infty$ (2)

(c)

x	0		$+\infty$
f'	‖	+	
f	‖	↗	$+\infty$
	$-\infty$		

(2) (*)

(d) f is continuous and strictly increases from $-\infty < 0$ to $+\infty > 0$ $\Rightarrow f(x) = 0$: unique root (2) (*)

(2) (*) $f(0.6) \times f(0.7) = -0.015 < 0$ verified

Now, $f(x) \le 0 \to x \le \alpha$ and $f(x) > 0 \to x > \alpha$ (2)

3) (a) $f''(x) = 2 - \frac{1}{x^2}$ $\quad f''(x) = 0$, $x^2 = \frac{1}{2}$, $x_I = \frac{\sqrt{2}}{2}$ only

$y_I = \frac{1}{2} - \frac{1}{2}\ln 2$

(2) (*) (C) has a point of inflection I around which $f''(x)$ changes its sign.

(b) Graph 6

B (T): $h(x) = \ln x$ $\quad D_h =]0, +\infty[$ 2

1) Let $A(x) = f(x) - h(x) = x^2 > 0$ since $x > 0 \Rightarrow$ (C) above (T)

2) $g(x) = x^2 + \ln^2 x$ $\quad D_g =]0, +\infty[$

(a) $g'(x) = 2x + \frac{2\ln x}{x} = \frac{2}{x}\left(x^2 + \ln x\right) = \frac{2}{x} f(x)$ 2

(b) $g'(x)$ take the same sign as $f(x)$. 2 (*)

$f(\alpha) = 0,\ \alpha^2 + \ln\alpha = 0,\ \ln\alpha = -\alpha^2$

$g(\alpha) = \alpha^2 + (\ln\alpha)^2 = \alpha^2 + (-\alpha^2)^2 = \alpha^2 + \alpha^4$

x	0		α		$+\infty$
g'	‖	$-$	0	$+$	
g	‖ $+\infty$	$\searrow$	min $\alpha^2 + \alpha^4$	$\nearrow$	$+\infty$

3) $M_0(\alpha, \ln\alpha) \in (T)$ $\quad M(x, \ln x) \in (T)$ 1

(a) $OM_0^2 = \alpha^2 + \ln^2\alpha = \alpha^2 + \alpha^4$ 1 $\quad OM^2 = x^2 + \ln^2 x$

(b) $OM_0^2 = g(\alpha) \rightarrow OM_0 = \sqrt{g_{min}} \leq \sqrt{g(x)} = OM$

Then $OM_0 \leq OM$ (*) 2

(c) slope tangent at M_0 $= h'(\alpha) = \frac{1}{\alpha}$ } Their product equal to -1

slope $OM_0 = \frac{\ln\alpha}{\alpha} = \frac{-\alpha^2}{\alpha} = -\alpha$ 2 (*) verified

V 1) $r = r\left(A, \frac{\pi}{2}\right)$ $h = h(E, 2)$

$f = r \circ h$

(a) Since f is the composition of a dilation and a rotation then it is a similitude. (2) (*)

$k_f = k_r \cdot k_h = 1(2) = 2$

$\alpha_f = \alpha_r + \alpha_h = \frac{\pi}{2} + 0 = \frac{\pi}{2} \ (2\pi)$

f
$O \to C$
$A \to D$
$\Omega \to \Omega$

(b) $f(O) = r \circ h(O) = r(B) = C$

$f(A) = r \circ h(A) = r(E) = D$

$(\vec{OA}, \vec{CD}) = \alpha_f = \frac{\pi}{2} \Rightarrow (OA) \perp (CD)$ (*)

2) (a) $k_s = \frac{AC}{AD} = \frac{1}{2}$ $\alpha_s = (\vec{AD}, \vec{CA}) = (\vec{AD}, \vec{AE}) = \frac{\pi}{2}$

s
$A \to C$
$D \to A$
$I \to I$

(b) $s[(CD)] = (AO)$ 2 Since $(CD) \perp (AO)$ and $s(D) = A$

$s[(AO)] = (CD)$ 2 Since $(AO) \perp (CD)$ and $s(A) = C$

(c) $s[(CD) \cap (AO)] = (AO) \cap (CD) \Rightarrow s(I) = I$ 2 (*)

3) $g = f \circ s$

(a) $\alpha_g = \alpha_f + \alpha_s = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$k_g = k_f \cdot k_s = 2\left(\frac{1}{2}\right) = 1$

$g(D) = f \circ s(D) = f(A) = D$

$g = S_D$: central symmetry of center D. 3 (*)

(b) $g(A) = f \circ s(A) = f(C)$ but $g(A) = S_D(A) = A'$

Then $f(C) = A'$ (*) 2

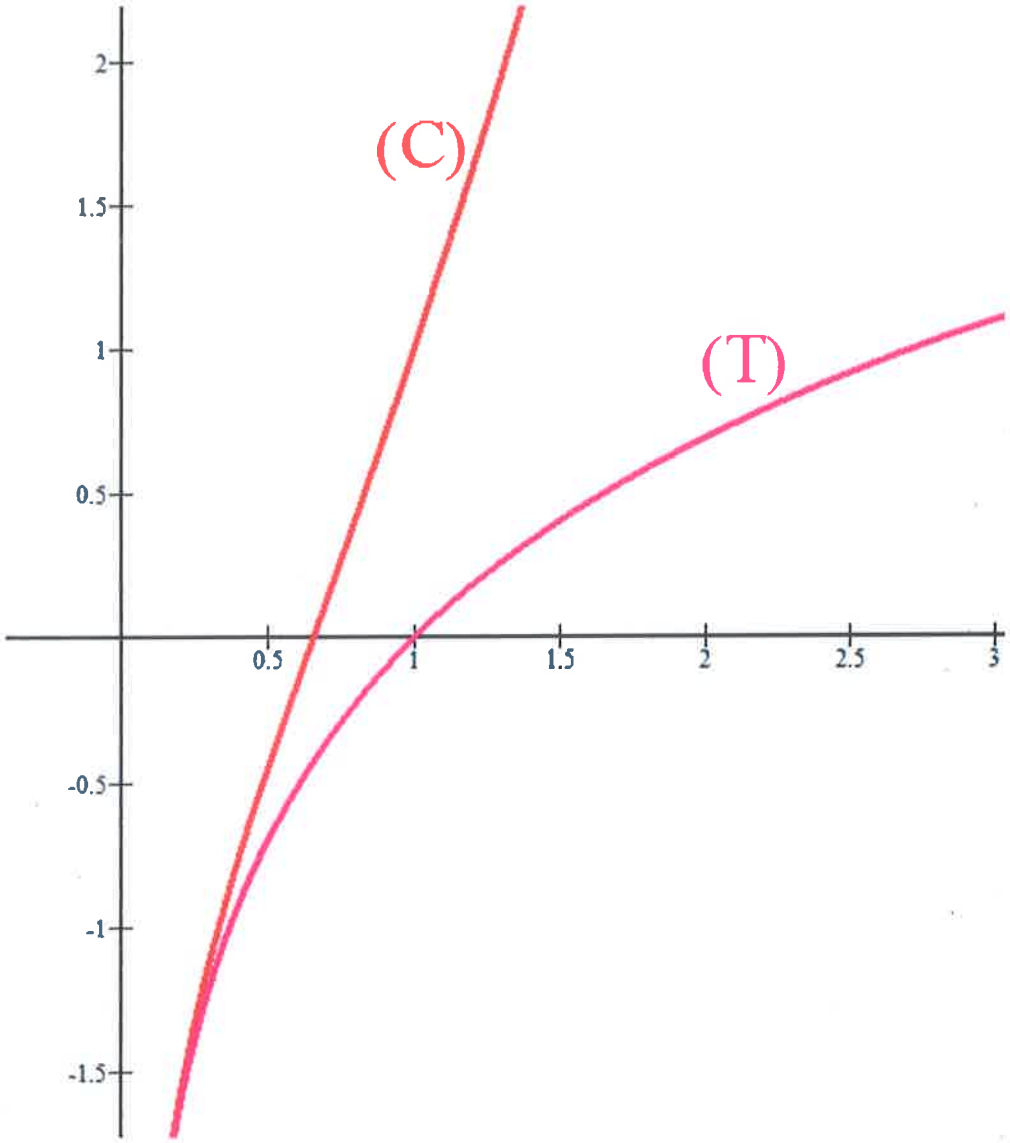
4) (a) $z_A = 0 \quad z_B = 1 \quad z_E = -2i \quad z_O = \frac{z_B + z_E}{2} = \frac{1}{2} - i$ 2

$z_C = i$ 1 $\quad z_{A'} = -4$ 1

(b) $z' = az + b \quad a = ke^{i\alpha} = 2e^{i\frac{\pi}{2}} = 2i$

$z_D = \cancel{a z_A}^{0} + b \Rightarrow b = -2 \Rightarrow z' = 2iz - 2$ 3

$z_\Omega = \frac{b}{1-a} = \frac{-2}{1-2i} \rightarrow z_\Omega = \frac{-2}{5} - \frac{4}{5}i$ 3


(C)
(T)
2
1.5
1
0.5
-0.5
-1
-1.5
0.5
1
1.5
2
2.5
3