This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls Loueizeh - Baabda

Stage Exam 03
10FEB2026

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	12M	
Teacher :	Bechara Kordahi	
Duration :	1 hour	
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(75 points)

Consider the function $g(x)=e^x-x$ defined over $\Re$.

1) Calculate $g'(x)$ and set-up the table of variations of g .

2) Deduce that $g(x)\geq 1$ for all values of x .

Consider another function $(C): f(x)=Ln\,(e^x-x)$ defined also over $\Re$.

Let $(d): y=x$.

a) Determine $\underset{x\to-\infty}{Lim}\ f(x)$ and calculate $f(-6)$.

b) Determine $\underset{x\to+\infty}{Lim}\ f(x)$. Then , prove that the line (d) is an asymptote to (C) .

c) Study , according to the values of x , the relative positions of (C) and (d) .

d) Calculate $f'(x)$ and set-up the table of variations of f .

e) Draw (C) and (d) .

Consider the function $t(x)=(2-x)e^x-1$

with its table of variations shown in the adjacent figure .

x	$-\infty$		1		$+\infty$
t'(x)		+	0	−	
t(x)	−1	↗	e − 1	↘	$-\infty$

f) The equation $t(x)=0$ has 2 roots α and β .
Prove that $-1.2<\alpha<-1.1$ and $1.8<\beta<1.9$.

g) Let $h(x)=Ln\,[(1-g(x)).t(x)]$. Determine the domain of definition of $h(x)$.

Question II-(25 points)

Consider the sequence (U_n) defined as : $\begin{cases} U_0=0.5 \\ U_{n+1}=\dfrac{2U_n}{1+U_n} \end{cases}$ $n\in N$

1) Prove that $0\leq U_n\leq 1$.

2) Show that $U_{n+1}-U_n=\dfrac{U_n(1-U_n)}{1+U_n}$. Deduce that the sequence (U_n) increases .

3) Prove that the sequence (U_n) converges . Find its limit L .

I $g(x) = e^x - x$ $D_g = R$

a) $g'(x) = e^x - 1$ 5

x	$-\infty$		0		$+\infty$
g'		$-$	0	$+$	
g	$+\infty$	$\searrow$	min	$\nearrow$	$+\infty$

5

b) $g_{min} = 1 \rightarrow g(x) \geq 1$ 5 1

(c): $f(x) = \ln(e^x - x)$ $D_f = R$ (d): $y = x$

a) $\lim_{x \to -\infty} f(x) = \ln(0 + \infty) = +\infty$ 5 $f(-6) = 1.8$ 5

b) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left[e^x \left(1 - \frac{x}{e^x}\right)\right] = +\infty$ (0) 5

$\lim_{x \to +\infty} [f(x) - y_{(d)}] = \lim_{x \to +\infty} [\ln(e^x - x) - \ln e^x] = \lim_{x \to +\infty} \ln\left(\frac{e^x - x}{e^x}\right)$ 5

$= \lim_{x \to +\infty} \ln\left(1 - \frac{x}{e^x}\right) = 0$ (0)

(d) asymptote to (c) as $x \to +\infty$

c) $f(x) - y_{(d)} = \ln\left(\frac{e^x - x}{e^x}\right)$ 5

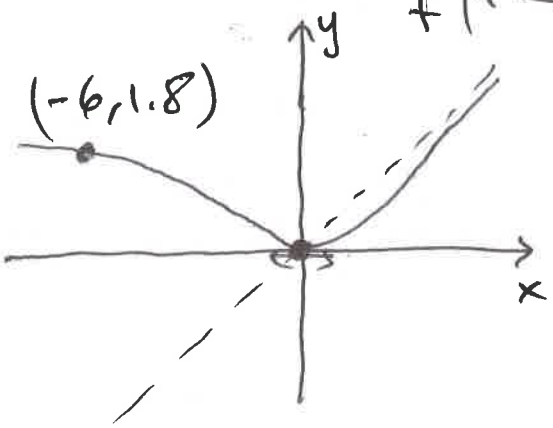
$x = 0$ (c) cuts (d) at 0
$x > 0$ (c) below (d)
$x < 0$ (c) above (d)

d) $f'(x) = \frac{e^x - 1}{e^x - x}$ 5

x	$-\infty$		0		$+\infty$
f'		$-$	0	$+$	
f	$+\infty$	$\searrow$	min	$\nearrow$	$+\infty$

5

e) $(-6, 1.8)$



10

I f) $t(x) = (2-x)e^x - 1$

$t(-1.2) \times t(-1.1) = -0.0012 \leq 0$ $\quad -1.2 < \alpha < -1.1$

10 $t(1.8) \times t(1.9) = -0.065 \leq 0$ $\quad 1.8 < \beta < 1.9$

g) $h(x) = \ln\left[(1-g(x)) \cdot t(x)\right]$

$[1-g(x)][t(x)] > 0$ $\quad 1-g(x) \neq 0$ since $g(x) \geq 1$

5 Then $t(x) < 0 \longrightarrow x < \alpha$ or $x > \beta$

II $\begin{cases} u_0 = 0.5 \\ u_{n+1} = \dfrac{2u_n}{1+u_n} \end{cases}$

a) $0 \leq u_0 = 0.5 \leq 1$ True for $n=0$

$u_{n+1} = f(u_n) = \dfrac{2u_n}{1+u_n}$

$f'(u_n) = \dfrac{2(1+u_n) - 1(2u_n)}{(1+u_n)^2} = \dfrac{2}{(1+u_n)^2} > 0$

5 $f(u_n)$ strictly increases

Then $f(0) \leq f(u_k) \leq f(1)$, $0 \leq u_{k+1} \leq 1$ verified

2) $u_{n+1} - u_n = \dfrac{2u_n}{1+u_n} - u_n = \dfrac{2u_n - u_n - u_n^2}{1+u_n} = \dfrac{u_n(1-u_n)}{1+u_n}$ 5

$0 \leq u_n \leq 1 \Leftarrow \begin{cases} u_n > 0 \\ 1-u_n > 0 \\ 1+u_n > 0 \end{cases}$ $\Big\}$ $u_{n+1} - u_n > 0$ 5

(u_n) increases

3) * (u_n) increases with 1 as an upper bound

Then (u_n) converges. 5

* $n \to +\infty$, $u_n \to l$ and $u_{n+1} \to l$ then: 5

$l = \dfrac{2l}{1+l}$ $\quad l \neq 0$ then $1+l = 2 \to l = 1$