This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Loueizeh - Baabda

Mock Exams : Pt2

13, 14, 15 JUN2022

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	12E
Teacher :	Bishara Kordahi
Duration :	1 hour 30 minutes

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(35 points)

Consider the function f defined on $[0,+\infty[$ as $f(x)=2x+1+xe^{-x+2}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

Let (d) be the line with equation $y = 2x + 1$.

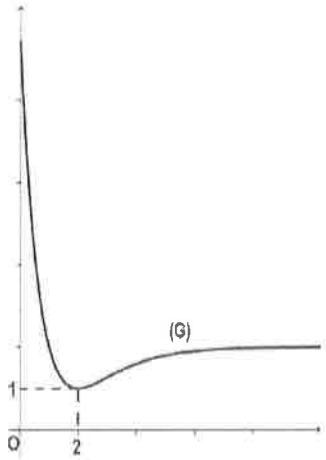
1) Determine $\lim\limits_{x\to+\infty} f(x)$ and calculate f(1).

2) a- Study, according to the values of x, the relative position of (C) and (d) and specify the coordinates of their point of intersection.
   b- Show that (d) is an asymptote to (C).

3) a- Show that $f'(x)=2+(1-x)e^{-x+2}$.
   b- The curve (G) of the function f' is shown in the adjacent figure.
   For all x on $[0,+\infty[$, verify that $f'(x)>0$.
   c- Set the table of variations of f.

4) The line (D) with equation $y = 4x$ intersects (C) at the point with abscissa α. Show that $1.66 < \alpha < 1.68$.

5) Draw (d), (D) and (C).



Part B

Determine the coordinates of the point K of (C) , the farthest from the line (d)

Question II-(30 points)

In a game we use:

- A fair die;
- An urn U that contains 4 white and 3 red balls;
- An urn V that contains 17 white and 18 red balls.

A- The die is rolled.

If the six appears then a ball is drawn at random from the urn U. otherwise a ball is drawn at random from the urn V.

1) Prove that the probability that the drawn ball is white and from urn U is equal to $\frac{2}{21}$.

2) Calculate the probability of drawing a white ball.

3) Knowing that the drawn ball is white, calculate the probability that it is drawn from urn V.

B- In this part a new game consists of drawing balls, randomly and successively, from the urn U without replacement. This game ends once a white ball is drawn.

1) Calculate the probability that this game ends at the third draw.

2) Determine the probability that this game ends at the first , second or fourth draw.

Question III-(35 points)

A- Given below the table of variations of a function f defined on $]1;+\infty[$ by:
$f(x)=3\ln(x-1)+ax+b$, where a and b are two real numbers.

x	1	2	$\frac{5}{2}$	$+\infty$
f '(x)	||	+	0	−
f(x)	||	3 ↗	$2+3\ln\frac{3}{2}$	↘
	||	$-\infty$		$-\infty$

Use the information in this table to find the values of a and b.

B- Suppose that f is defined on $]1;+\infty[$ by $f(x)=3\ln(x-1)-2x+7$, and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Write an equation of the tangent (T) to the curve (C) at the point of abscissa 2.
2) Draw (T) and (C).
3) The line with equation $y = 0.8x$ intersects (C) in two points. Show that the abscissa α of one of them is such that $3.4 < \alpha < 3.5$.
4) Prove that the equation $f(x)=0$ has two roots 'a' and 'b' such that :

$$1.21 < a < 1.22 \quad \text{and} \quad 5.87 < b < 5.88$$

5) Does (C) have an asymptote or an asymptotic direction ?

Justify fully your answer especially if it has an asymptotic direction .

Mock Part 2 Mathematics

(I) (c): $f(x) = 2x+1+xe^{-x+2}$ $D_f = [0, +\infty[$

(d): $y = 2x+1$

1) $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left(2x+1+\frac{x}{e^{x-2}}\right) = \lim\limits_{x\to+\infty}\left(2x+1+\frac{1}{e^{x-2}}\right) = +\infty$ 3

$f(1) = 3+e \approx 5,7$ 2

2) (a) Let $g(x) = f(x) - (2x+1) = \frac{x}{e^{x-2}}$

5 (*) $g(x) > 0$ (c) above (d) for $x > 0$

$g(x) = 0$ (c) cuts (d) at $x = 0, y = 1$

(b) $\lim\limits_{x\to+\infty}[f(x)-(2x+1)] = \lim\limits_{x\to+\infty}\frac{x}{e^{x-2}} = 0$ verified (*) 2

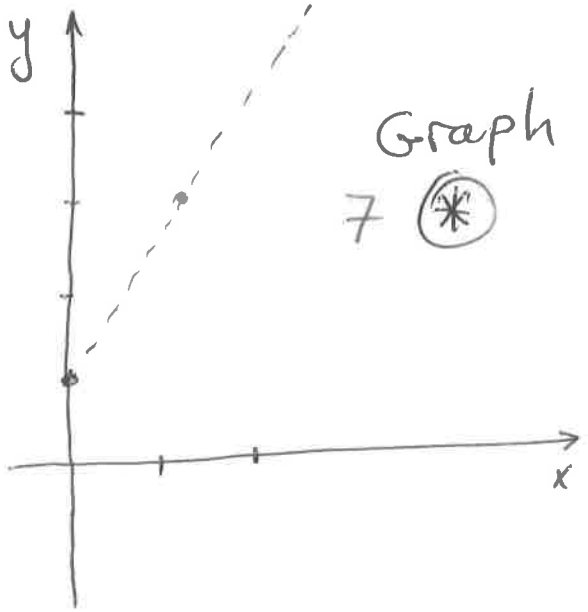
3) (a) $f'(x) = 2+1.e^{-x+2}+e^{-x+2}(-1)(x) = 2+(1-x)e^{-x+2}$ verified (*) 3

(b) $f'_{min} = 1 > 0 \Rightarrow f'(x) > 0$ for all $x \geq 0$ (*) 2

(c)

x	0		$+\infty$
f'		+	
f	1	↗	$+\infty$

5 (*)



Graph 7 (*)

4) Let $h(x) = f(x) - 4x$

$h(1.66) \times h(1.68) = -0.0005 < 0$

verified (*) 3

Part B $f'(x_K) = \text{slope}_{(d)} \rightarrow 2+(1-x)e^{2-x} = 2 \Rightarrow 1-x_K = 0$

$x_K = 1$, $y_K = f(1) = 3+e$ $K(1, 3+e)$ 3

II

die

4W 3R	17W 18R
Urn U	Urn V

A 1) $P(W \cap U) = P(U) \cdot P(W/U) = \frac{1}{6} \cdot \frac{4}{7} = \frac{2}{21}$ 4

2) $P(W) = P(W \cap U) + P(W \cap V) = \frac{2}{21} + \frac{5}{6} \cdot \frac{17}{35} = \boxed{\frac{1}{2}}$ 5

3) $P(V/W) = \frac{P(V \cap W)}{P(W)} = \frac{\frac{5}{6} \cdot \frac{17}{35}}{\frac{1}{2}} = \frac{17}{21}$ 5

B 1) $P_3 = \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{4}{5} = \frac{4}{35} = P(R_1 R_2 W_3)$ 4

2) $P_1 = \frac{4}{7} = P(W_1)$ 4 $\quad P_2 = \frac{3}{7} \cdot \frac{4}{6} = \frac{2}{7} = P(R_1 W_2)$ 4

$P_4 = \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{4}{4} = \frac{1}{35} = P(R_1 R_2 R_3 W_4)$ 4

III A $f(x) = 3\ln(x-1) + ax + b \qquad f(2) = 3, \; 2a + b = 3$

$f(5/2) = 2 + 3\ln\frac{3}{2} \rightarrow 3\ln\frac{3}{2} + \frac{5}{2}a + b = 2 + 3\ln\frac{3}{2}$

$\begin{cases} \frac{5}{2}a + b = 2 \\ 2a + b = 3 \end{cases}$ $\quad \frac{a}{2} = -1$ $\quad a = -2, \; -4 + b = 3, \; b = 7$ 8

Note One can use $f'(\frac{5}{2}) = 0 \qquad f'(x) = \frac{3}{x-1} + a$

then $\frac{3}{\frac{5}{2} - 1} + a = 0 \rightarrow a = -2$ and $b = 7$

B(c): $f(x) = 3\ln(x-1) - 2x + 7$

1) $x = 2, y = 3$ $\quad f'(x) = \frac{3}{x-1} - 2$ $\quad f'(2) = 1$

$y - 3 = 1(x-2) = x - 2 \rightarrow$ (T): $y = x + 1$ 5

3) $y = 0.8x$ Let $g(x) = f(x) - 0.8x = 3\ln(x-1) - 2.8x + 7$

$g(3.4) \times g(3.5) = -0.0054 < 0$ verified (*) 3

4) $f(x) = 0$ $\quad f(1.21) \times f(1.22) = -0.0018 < 0$ (*) 3

$f(5.87) \times f(5.88) = -0.000042 < 0$ (*) 3 verified

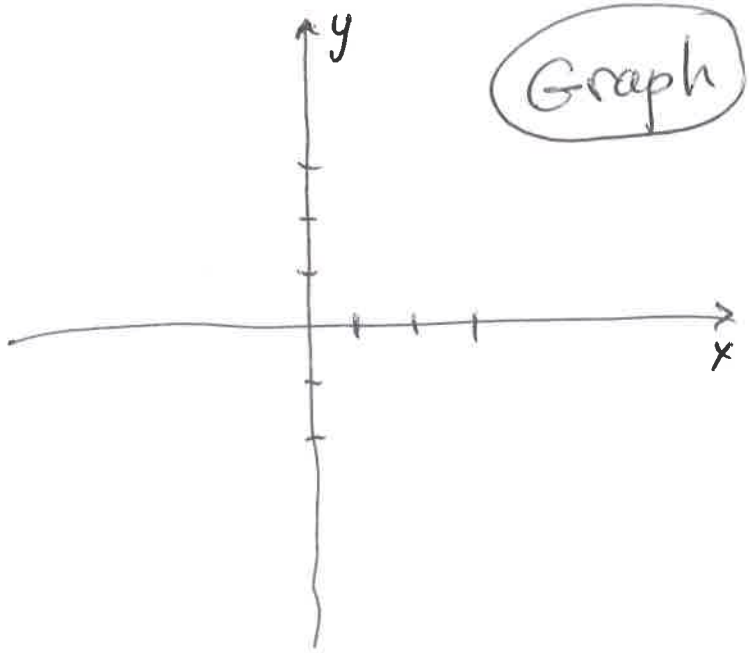
5) $a = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \left[3\frac{\ln(x-1)}{x} - 2 + \frac{7}{x} \right]$

$= \lim_{x \to +\infty} \left(\frac{3}{\frac{x-1}{1}} - 2 + \frac{7}{x} \right) = 0 - 2 + 0 = -2$ 7

$b = \lim_{x \to +\infty} [f(x) + 2x] = \lim_{x \to +\infty} (3\ln(x-1) + 7) = +\infty + 7 = +\infty$

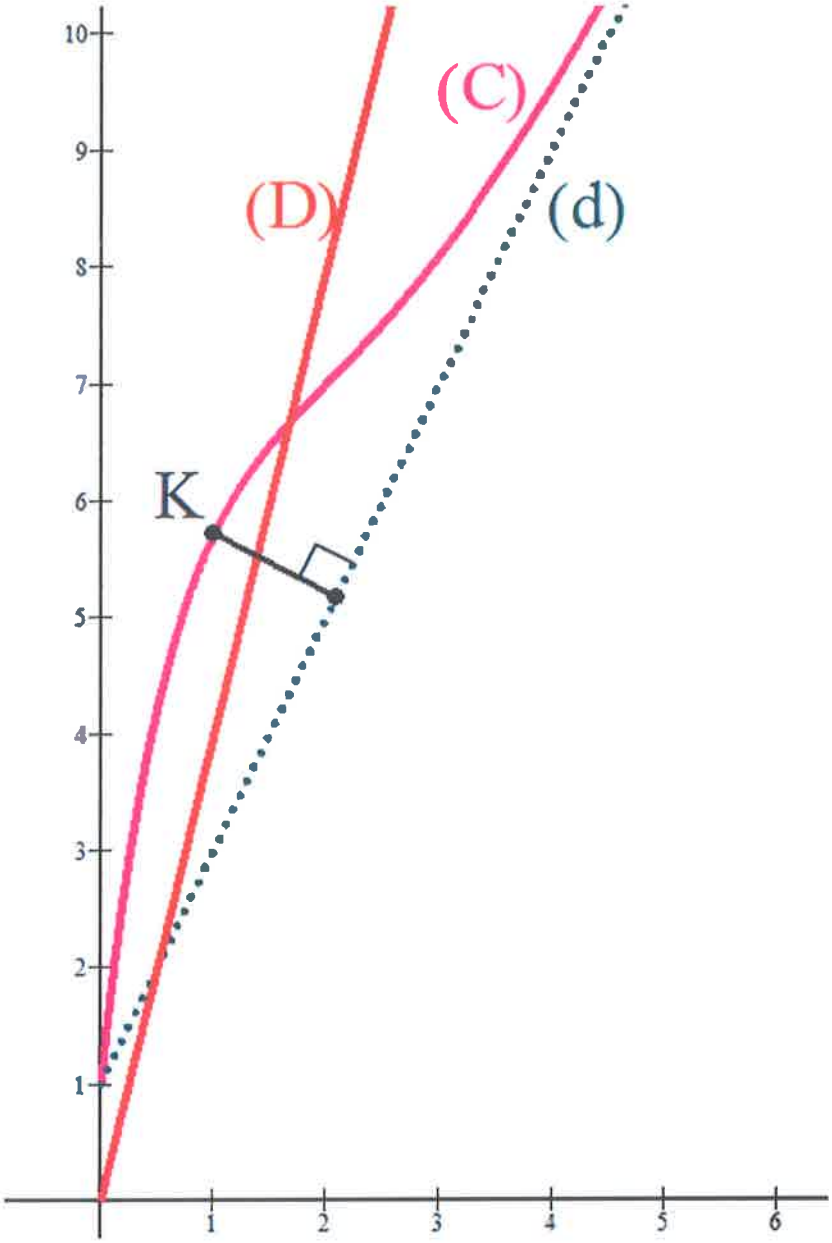
(c) has an asymptotic direction // $y = -2x$.

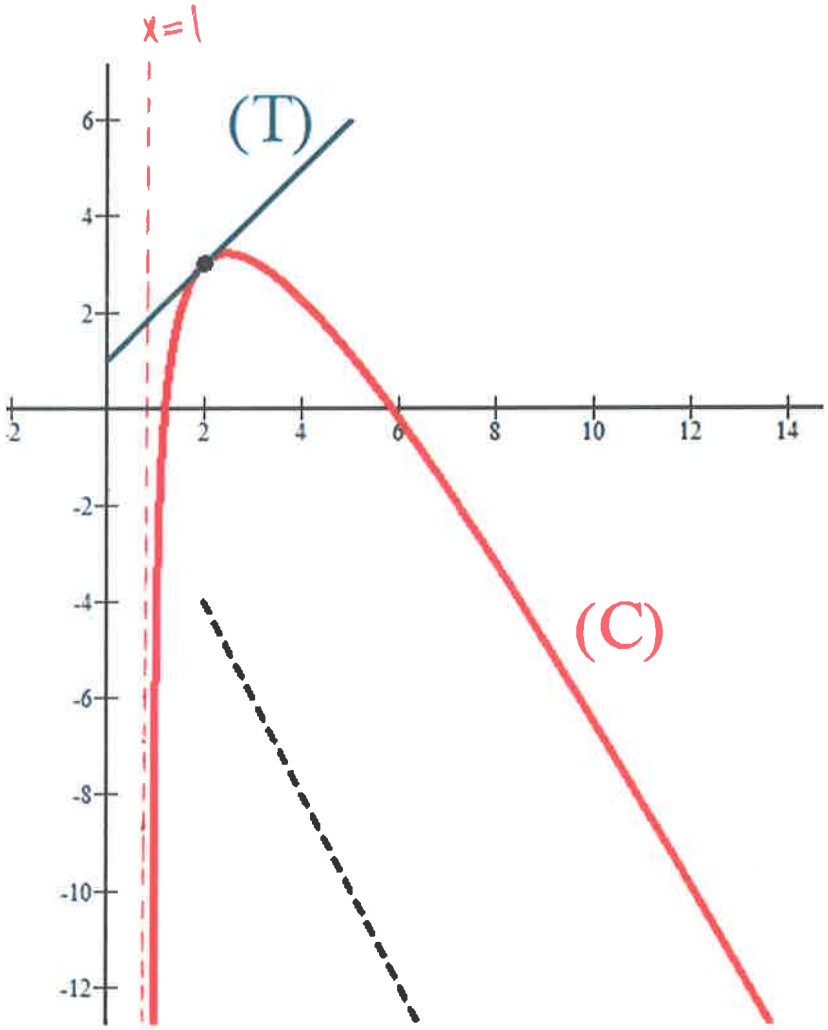
2)



Graph 6

3/3


(C)
(D)
(d)
K
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6


x=1
(T)
(C)
6
4
2
-2
-4
-6
-8
-10
-12
2
4
6
8
10
12
14