This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 01

06NOV2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	12M	
Teacher :	Bishara Kordahi	
Duration :	2 hours	
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(40 points)

Consider the direct rectangle ABCD where AB = 4 and AD = 3.

Let H be the orthogonal projection of A on (BD) and h be the dilation, of center H ,

that transforms D to B.

a- Determine the image of the straight line (AD) by h.

b- Deduce the image E of point A by h. Plot E.

c- Construct the point F image of B by h and the point G image of C by h, then determine the image of the rectangle ABCD by h.

Question II-(40 points)

Point $M(x,y)$ is transformed to point $M'(x',y')$ such that : $\begin{cases} x' = -2x + 4 \\ y' = -2y - 6 \end{cases}$.

a- Prove that the above transformation is that of a dilation whose center and ratio are to be found

b- Find the transform of the straight line (D) : $y = 4x - 2$.

c- (C) is the trigonometric circle which is the transform of another circle (I) . Determine (I) .

d- There is another dilation that takes circle (I) to circle (C) . Find its properties .

Question III-(40 points)

Consider the following function $f(x) = x - 3 - \dfrac{3Lnx}{1 - Lnx}$ whose graph is (C) .

1) Determine its domain of definition .

2) Find the limits of $f(x)$ at the boundaries of its domain .

3) Prove that (C) admits 2 asymptotes to be determined with proof .

4) Find $f'(x)$.

Question IV-(40 points)

1) Determine : $\lim\limits_{x\to 1^+} \dfrac{Ln(x-1)}{x-1}$ and $\lim\limits_{x\to 0} \dfrac{Ln(\cos x)}{1-\cos x}$.

2) Find the derivative of $f(x)=Ln\left(\dfrac{1-\sin x}{1+\sin x}\right)$.

3) Determine the domain of definition of $f(x)=Ln\,(Lnx)$

4) Solve the equation : $|Lnx|=-Lnx$.

5) Solve the following system : $\begin{cases} y-x=\dfrac{2}{e} \\ Lnx^2=-1+Lny \end{cases}$

6) Solve the equation in x : $Ln(x-1)+Ln(x+1)=3Ln2$

SE01 — 12M Maths correction — 6/11/2023

I a) $h[(AD)] = (BC)$ since $h(D) = B$ and $(AD) \nparallel (BC)$ 8

b) $\{E\} = (BC) \cap (AH)$ 8

c) $h[(AB)] = (EF)$ since $h(A) = E$ and $(AB) \nparallel (EF)$

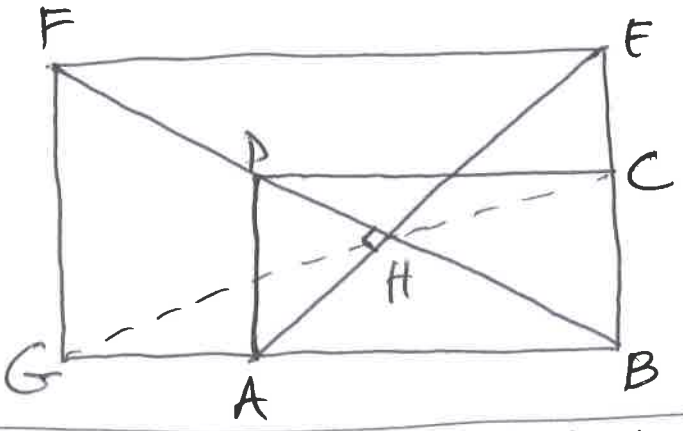
$\{F\} = (BH) \cap (EF)$ 8

$h[(BC)] = (FG)$ since $h(B) = F$ and $(BC) \nparallel (FG)$

$\{G\} = (CH) \cap (FG)$ 8 $h(ABCD) = EFGB$ 8

direct rectangles

h
$H \to H$
$D \to B$
$A \to E$
$B \to F$
$C \to G$



II $M(x,y) \to M'(x',y')$ $\begin{cases} x' = -2x + 4 \\ y' = -2y - 6 \end{cases}$

a) $x' = Kx + (1-K)a$

$y' = Ky + (1-K)b$

$K = -2$ $\begin{cases} 3a = 4 \to a = \frac{4}{3} \\ 3b = -6 \to b = -2 \end{cases}$ 10

It's a dilation $h(I, -2)$ where $I\left(\frac{4}{3}, -2\right)$

b) $x = \frac{4 - x'}{2}$ $y = \frac{-6 - y'}{2}$ $(D): y = 4x - 2 \to (D')$

where $\frac{-6-y'}{2} = 4\left(\frac{4-x'}{2}\right) - 2$, $-6 - y' = 16 - 4x' - 4$ 10

Then $(D'): y' = 4x' - 18$ or $y = 4x - 18$

c) $(I) \to (C) : x'^2 + y'^2 = 1 \to (-2x+4)^2 + (-2y-6)^2 = 1$ 10

$4(x-2)^2 + 4(y+3)^2 = 1 \to \boxed{(I) : (x-2)^2 + (y+3)^2 = \frac{1}{4}}$

Then (I) is the circle of center $(2,-3)$ $R = 1/2$

d) The other dilation $\boxed{h' = h(J, 2)}$ 5 where

$\vec{JO} = 2\vec{JO'}$ $\quad O(0,0)$ $\quad O'(2,-3)$

$x_O - x_J = 2(x_{O'} - x_J) = 2x_{O'} - 2x_J$, $x_J = 2x_{O'}$

and $y_J = 2y_{O'}$ Hence $\boxed{J(4,-6)}$ 5

III (c): $f(x) = x - 3 - \frac{3\ln x}{1-\ln x} = x + \frac{-3+3\ln x - 3\ln x}{1-\ln x} = x - \frac{3}{1-\ln x}$

1) $\left.\begin{array}{l} x > 0 \\ \ln x \neq 1,\ x \neq e \end{array}\right\}$ $\boxed{D_f =]0, e[\cup]e, +\infty[}$ 10

2) $\lim_{x \to 0^+} f(x) = 0 - 3 + \frac{\infty}{\infty} = \lim_{x \to 0^+} \left(x - 3 - \frac{\frac{3}{x}}{\frac{-1}{x}} \right) = 0 - 3 + 3 = 0$ 4

$\lim_{x \to e} f(x) = e - 3 - \frac{3}{0} = \boxed{\pm\infty}$ 3

$\lim_{x \to +\infty} f(x) = +\infty - \frac{3}{-\infty} = \boxed{+\infty}$ 3

3) $\lim_{x \to e} f(x) = \pm\infty$ then $\boxed{x = e \ \text{VA}}$ 5

$\lim_{x \to +\infty} [f(x) - x] = \lim_{x \to +\infty} \frac{-3}{1-\ln x} = 0^+$ then $\boxed{y = x \ \text{OA}}$ 5

4) $f'(x) = 1 + \frac{-3/x}{(1-\ln x)^2} = \boxed{1 - \frac{3}{x(1-\ln x)^2}}$ 10

IV

1) $\lim_{x\to 1^+} \frac{\ln(x-1)}{x-1} = \frac{-\infty}{0^+} = -\infty$ 3

$\lim_{x\to 0} \frac{\ln(\cos x)}{1-\cos x} = \lim_{x\to 0} \frac{\frac{-\sin x}{\cos x}}{\frac{\sin x}{1}} = \lim_{x\to 0}\left(\frac{-1}{\cos x}\right) = -1$ 3

2) $f(x) = \ln\left(\frac{1-\sin x}{1+\sin x}\right)$, $f'(x) = \frac{\frac{-\cos x(1+\sin x)-\cos x(1-\sin x)}{(1+\sin x)^2}}{\frac{1-\sin x}{1+\sin x}}$ 10

$f'(x) = \frac{-2\cos x}{1-\sin^2 x} = \frac{-2\cos x}{\cos^2 x} \rightarrow f'(x) = \frac{-2}{\cos x}$

3) $f(x) = \ln(\ln x)$ $\left.\begin{matrix} x>0 \\ \ln x>0,\ x>1 \end{matrix}\right\}$ $D_f =]1, +\infty[$ 4

4) $|\ln x| = -\ln x$ $x>0$ $\ln x<0,\ x<1$

Then $0<x<1$ 5

5) $\begin{cases} y-x=\frac{2}{e} \\ \ln x^2 = -1+\ln y \end{cases}$ $x^2>0,\ x\neq 0$ and $y>0$

$\ln x^2 + 1 = \ln y = \ln x^2 + \ln e$

$\ln y = \ln(ex^2) \rightarrow y = ex^2$ then $ex^2 - x - \frac{2}{e} = 0$

or $e^2x^2 - ex - 2 = 0$ $\Delta = e^2 + 8e^2 = 9e^2$

$x = \frac{e \pm 3e}{2e^2}$ $\begin{cases} x_1 = 2/e, & y_1 = 4/e \\ x_2 = -1/e, & y_2 = 1/e \end{cases}$

Hence $(x,y) = \left\{\left(\frac{2}{e}, \frac{4}{e}\right), \left(\frac{-1}{e}, \frac{1}{e}\right)\right\}$ 10

6) $\ln(x-1) + \ln(x+1) = 3\ln 2$ $\left.\begin{matrix} x>1 \\ x>-1 \end{matrix}\right\}$ $x>1$

$\ln[(x-1)(x+1)] = \ln 2^3 = \ln 8$

$\ln(x^2-1) = \ln 8$, $x^2-1=8$, $x^2=9$, $x=\pm 3$

Hence $x=3$ only 5