This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School

for Boys and Girls — Loueizeh - Baabda

Stage Exam 06

19, 20, 21JUN2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	12E
Teacher :	Bishara Kordahi
Duration :	1 hour 30 minutes

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(30 points)

Part A

Consider two urns U and V where :

- U contains five balls : three red and two black .
- V contains three balls : one red and two black .

One ball is selected randomly from the urn U and two balls are selected randomly and simultaneously from the urn V .

1) Verify that the total number of all possible selections is 15 .

2) Calculate the number of selections consisting of one red ball from U and two balls of different colors from V .

3) Calculate the number of selections of three balls with the same color .

Part B

Answer , **with justification** , by **true** or **false** .

1) The equation $Ln(-x) = 2$ has no solution .

2) For x > 0 , if $g(x) = Ln(x^2 + x)$ then $g'(x) = \dfrac{1}{x^2 + x}$.

3) When $h(x) = Ln(1+x) + Ln(x-2)$, then $D_h =]-\infty, -1[\cup]2, +\infty[$.

Question II-(20 points)

Consider two urns U and V : **each** contains 10 balls .

U contains : 6 red and 4 yellow . V contains : 5 red , 4 black and 1 green .
A biased (fake) coin is given with the probability of Head is 3 times that of Tail .
The coin is flipped .

- If Tail shows up , 2 balls are selected randomly and simultaneously from U .
- If Head shows up , 2 balls are selected successively with replacement from V .

Consider the following events : U : "The selected urn is U"
R : "The selected balls are red" .

1) Show that $P(U) = \dfrac{1}{4}$.

2) Calculate $P(R/U)$ and $P(R \cap U)$. Deduce that $P(R) = \dfrac{13}{48}$.

3) The two selected balls are red . Find the probability that they are from urn U .

4) Determine the probability of selecting no red ball . Is it true to say that the probability of selecting exactly 1 red ball is more than 50% ? Explain.

Question-III (50 points)

Part A

Let g be the function defined over $\Re$ as : $g(x) = (1-x)e^{-x} + 1$

1) Find the limits of $g(x)$ at the bounds of its domain .

2) a- Set-up the table of variations of g .

b- Deduce that $g(x) > 0$ for all $x \in \Re$.

Part B

Now , consider the function $(C) : f(x) = xe^{-x} + x$ defined also over $\Re$.

1) a- Determine the limits of $f(x)$ at the bounds of its domain .

b- Show that the straight line $(L) : y = x$ is an asymptote to (C) .

c- Discuss , according to the values of x , the relative positions of (C) and (L) .

2) a- Verify that $f'(x) = g(x)$. Then , set-up the table of variations of f .

b- Discuss , according to the values of x , the concavity of (C) .

c- Determine the point E on (C) where the tangent (T) to (C) is parallel to (L) .

d- Plot (L) , (T) and (C) on the same system of axes . **(use 1 unit = 2cm)**

Note : You MUST draw your graphs on graph sheets otherwise will NOT be corrected .

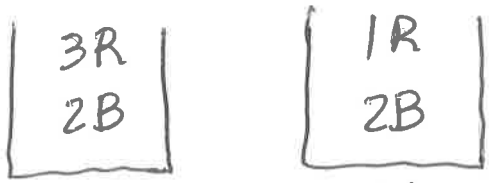
3) Consider the function h defined as $h(x) = Ln[y_E - f(x)]$.

a- Determine the domain of definition of h .

b- Set-up the table of variations of h .

12E Mathematics Mock June, 2023

(I) A



1 ball from U
2 balls from V simultaneously

1) $N_1 = {}_5C_1 \times {}_3C_2 = 15$ selections 5

2) $N_2 = {}_3C_1 \cdot {}_1C_1 \times {}_2C_1 = 6$ selections 5

3) $N_3 = {}_2C_1 \times {}_2C_2 = 2$ selections 5

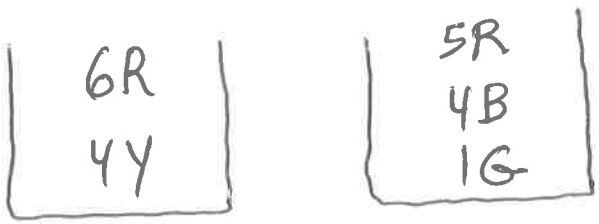
B 1) $\ln(-x) = 2 \rightarrow -x = e^2 \rightarrow x = -e^2$ False 5

2) $g(x) = \ln(x^2+x) \rightarrow g'(x) = \dfrac{2x+1}{x^2+x}$ False 5

3) $h(x) = \ln(1+x) + \ln(x-2)$ $1+x>0$, $x>-1$
$x-2>0$, $x>2$ 5

then $x>2$ False

(II)



Tail: 2 balls from U simultaneously

Head: 2 balls from V successively with replacement

1) $P(H) = 3P(T)$ $P(H) + P(T) = 1$, $3P(T) + P(T) = 1$

$4P(T) = 1$ $P(T) = P(U) = \frac{1}{4}$ 5 and $P(H) = P(\bar{U}) = \frac{3}{4}$

2) $P(R/U) = \dfrac{{}_6C_2}{{}_{10}C_2} = \frac{1}{3}$ 2 $P(R \cap U) = P(R/U) \cdot P(U) = \frac{1}{12}$ 2

$P(R/\bar{U}) = \frac{5}{10} \cdot \frac{5}{10} = \frac{1}{4}$ $P(R \cap \bar{U}) = P(R/\bar{U}) \cdot P(\bar{U}) = \frac{3}{16}$

$P(R) = P(R \cap U) + P(R \cap \bar{U}) = \frac{13}{48}$ 4

3) $P(U/\bar{R}) = \dfrac{P(U \cap R)}{P(R)} = \dfrac{1/12}{13/48} = \dfrac{4}{13}$ 2

4) $P(\bar{R}/U) = \dfrac{{}_4C_2}{{}_{10}C_2} = \dfrac{2}{15}$ $\quad P(\bar{R} \cap U) = \dfrac{2}{15} \cdot \dfrac{1}{4} = \dfrac{1}{30}$

$P(\bar{R}/\bar{U}) = \dfrac{5}{10} \cdot \dfrac{5}{10} = \dfrac{1}{4}$ $\quad P(\bar{R} \cap \bar{U}) = \dfrac{1}{4} \cdot \dfrac{3}{4} = \dfrac{3}{16}$

$P(\bar{R}) = \dfrac{1}{30} + \dfrac{3}{16} = \dfrac{53}{240}$ 3

Hence $P(1 \text{ Red ball}) = 1 - \left(\dfrac{13}{48} + \dfrac{53}{240}\right) = \dfrac{61}{120} > \dfrac{1}{2}$ 2 TRUE

III A $\quad g(x) = (1-x)e^{-x} + 1 \qquad D_g = \mathbb{R}$

1) $\lim\limits_{x \to -\infty} g(x) = +\infty\,(+\infty) + 1 = +\infty$ 2

$\lim\limits_{x \to +\infty} g(x) = \lim\limits_{x \to +\infty} \left(\dfrac{1-x}{e^x} + 1\right) = \lim\limits_{x \to +\infty} \left(\dfrac{-1}{e^x} + 1\right) = 0 + 1 = 1$ 2 2

2) (a) $g'(x) = -e^{-x} - e^{-x}(-x) = (x-2)e^{-x}$

x	$-\infty$		2		$+\infty$
g'		$-$	0	$+$	
g	$+\infty$	↘	$\min$ $1-e^{-2}$	↗	1

4

(b) $g_{min} = 1 - e^{-2} \sim 0.86 > 0$

then $g(x) > 0$ for all $x \in D_g$ 2

B (c): $f(x) = xe^{-x} + x = x(e^{-x} + 1) \qquad D_f = \mathbb{R}$ 2

1) (a) $\lim\limits_{x \to -\infty} f(x) = -\infty\,(+\infty) = -\infty$

$\lim\limits_{x \to +\infty} f(x) = +\infty(0+1) = +\infty$ 2

(b) $\lim\limits_{x \to +\infty} [f(x) - x] = \lim\limits_{x \to +\infty} \dfrac{x}{e^x} = \lim\limits_{x \to +\infty} \dfrac{1}{e^x} = 0^+$ 4

$y = x$ OA to (C) at $x \to +\infty$

(c) $f(x) - x = \frac{x}{e^x}$

(c) cut (L) at (0,0)

(c) above (L) for $x > 0$

(c) below (L) for $x < 0$ 5

2) (a) $f'(x) = 1(e^{-x}+1) - e^{-x}.x = (1-x)e^{-x} + 1 = g(x) > 0$ 2

x	$-\infty$		$+\infty$
f'		+	
f	$-\infty$	↗	$+\infty$

3

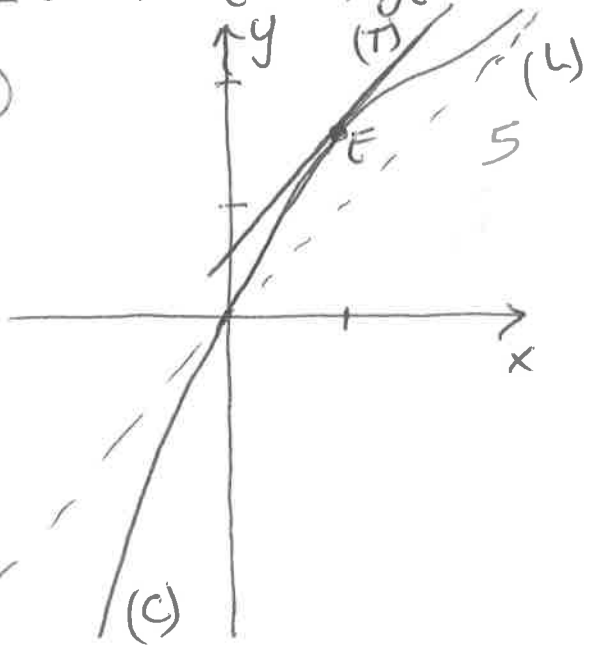
(b) $f''(x) = g'(x) = 0$ for $I(2, 2+2e^{-2})$

$f''(x) > 0$ for $x > 2$ 3 upward concavity

$f''(x) < 0$ for $x < 2$ downward concavity 3

(c) $f'(x_E) = 1 = g(x_E) \rightarrow \frac{1-x}{e^x} = 0 \rightarrow x_E = 1, y_E = 1+e^{-1}$

(d)



3) $h(x) = \ln[y_E - f(x)]$

(a) $y_E - f(x) > 0$ or $f(x) < y_E$

then $x < x_E \Rightarrow x < 1$

$D_h =]-\infty, 1[$ 4

(b) $h'(x) = \frac{-f'(x)}{y_E - f(x)} < 0$

x	$-\infty$		1
h'		−	
h	$+\infty$	↘	$-\infty$

5