This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School
for Boys and Girls — Loueizeh - Baabda

Stage Exam 04

20-25JUN2025

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	11S
Teacher :	Taline Hawatian
Duration :	1 hour
You are to answer the questions to this exam on : Squared paper	

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I.-(7 points)

In the space of a direct orthonormal system $(O;\ \vec{\imath};\ \vec{\jmath};\ \vec{k})$, consider the points $A(-1;3;3)$, $B(-6;1;1)$, , $C(4;\ -3;1), D(-1;\ -5;\ -1)$.

1) Calculate $\overrightarrow{AB}.\overrightarrow{AD}$ and deduce the measure of the angle $\widehat{BAD}$ to the nearest degree.
2) Show that B, C, and D form a plane.
3) Calculate the coordinates of the vector $\overrightarrow{CB} \times \overrightarrow{CD}$. Calculate the area of the triangle BCD.
4) Calculate the distance from point D to the straight line (BC).
5) Given the point $H(1;\ 8;-7)$, show that $\overrightarrow{AH}$ is perpendicular to the plane (BCD).

II.- (7 points)

In the complex plane, consider the points M and M' of affix $z = x + iy$ and $z' = x' + iy'$ respectively such that $z' = \frac{z-2}{z+i}$, $z \neq -i$.

1) Solve the equation $\frac{z-2}{z+i} = z - i$.
2) Show that $x' = \frac{x^2-2x+y^2+y}{x^2+(y+1)^2}$ and express y' in terms of x and y.
3) Determine the set of points M, in each of the following cases:
   a) z' is real.
   b) z' is pure imaginary.

III.- (7 points)

Calculate the following integrals:

1) $\int \frac{x-9}{\sqrt{x}-3} dx$ 2) $\int \frac{2x^3-9x^2+12x-2}{(x-2)^2} dx$ 3) $\int \frac{x+1}{\sqrt{3x-1}} dx$

IV.- (4 points)

1) Evaluate the sum S= $-2 + 1 - \frac{1}{2} + \cdots .. + \frac{1}{2^{16}}$ of geometric sequence (u_n).
2) The product of u_4 and u_6 of a geometric sequence is 4096. Evaluate u_5.
   If $u_3 = 1024$, calculate the first term u_1 and the common ratio r of the geometric sequence.

SE04/2024 2025/ Maths/ 11S

I.- $A(-1;3;3)$ $B(-6;1;1)$ $C(4;-3;1)$ $D(-1;-5;-1)$

1) $\vec{AB}(-5;-2;-2)$ $\vec{AD}(0;-8;-4)$

$\vec{AB}.\vec{AD} = (-5)(0) + (-2)(-8) + (-2)(-4) = 24$ 1

$\cos \hat{BAD} = \dfrac{\vec{AB}\cdot\vec{AD}}{\|\vec{AB}\| \times \|\vec{AD}\|} = \dfrac{24}{\sqrt{25+4+4}\cdot\sqrt{(-8)^2+(-4)^2}}$

$= \dfrac{24}{\sqrt{33}\cdot\sqrt{80}} = 0.467.$ 0.5

$\hat{BAD} = 62°$ 0.5

2) $\vec{BC}(10;-4;0)$ $\vec{CD}(-5;-2;-2)$

$\dfrac{10}{-5} \neq \dfrac{-4}{-2}$ $\Rightarrow$ $\vec{BC}$ and $\vec{CD}$ are not collinear vectors. 0.5

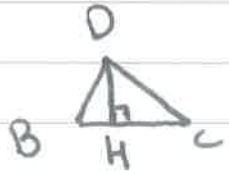
Therefore B, C, D form a plane.

3) $\vec{CB} \times \vec{CD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -10 & 4 & 0 \\ -5 & -2 & -2 \end{vmatrix} = -8\vec{i} - 20\vec{j} + 40\vec{k}$ 1.5

$\mathcal{A}_{\Delta BCD} = \dfrac{\|\vec{CB} \times \vec{CD}\|}{2} = \dfrac{\sqrt{64+400+1600}}{2} = 2\sqrt{129}\ u^2$ 1

4) $\mathcal{A}_{BCD} = \dfrac{DH \times BC}{2}$ $\|\vec{BC}\| = \sqrt{10^2+(-4)^2} = \sqrt{116} = 2\sqrt{29}$

$DH = \dfrac{2(2\sqrt{129})}{2\sqrt{29}} = 4.22\ u$ 1



5) $\vec{n} = \vec{CB} \times \vec{CD} = -8\vec{i} - 20\vec{j} + 40\vec{k}$

$\vec{AH}(2; 5; -10)$

$\vec{AH} = -\frac{1}{4}\vec{n}$ 1

$\vec{AH}$ and $\vec{n}$ are collinear vectors.

Therefore $\vec{AH}$ is perpendicular to plane (BCD).

II- $z' = \frac{z-2}{z+i}$ $\quad z \neq -i$

1) $\frac{z-2}{z+i} = z - i$

$z - 2 = (z-i)(z+i)$ 1

$z - 2 = z^2 - i^2$

$z^2 - z + 3 = 0$ $\quad \Delta = 1 - 12 = -11 = 11i^2$

$z_1 = \frac{1 + i\sqrt{11}}{2}$ 0.5 $\quad z_2 = \frac{1 - i\sqrt{11}}{2}$

2) $z' = x' + iy'$

$= \frac{x + iy - 2}{x + iy + i}$

$= \frac{(x-2) + iy}{x + (y+1)i} \cdot \frac{x - (y+1)i}{x - (y+1)i}$ 0.5

$= \frac{x(x-2) - (x-2)(y+1)i + xyi + y(y+1)}{x^2 + (y+1)^2}$

$= \frac{x^2 - 2x - xyi - xi + 2yi + 2i + xyi + y^2 + y}{x^2 + (y+1)^2}$ 1

$= \frac{x^2 - 2x + y^2 + y}{x^2 + (y+1)^2} + \frac{(-x + 2y + 2)}{x^2 + (y+1)^2} i$

$x' = \frac{x^2 - 2x + y^2 + y}{x^2 + (y+1)^2}$ $\quad y' = \frac{-x + 2y + 2}{x^2 + (y+1)^2}$ 1

3) a) z' is real. $\text{Im}(z')=0$ $\quad y'=0$

$$\frac{-x+2y+2}{x^2+(y+1)^2}=0$$

$-x+2y+2=0$ and $x^2+(y+1)^2\neq 0$ 1.5

The set of points M is the straight line of equation $-x+2y+2=0$ excluding the point $(0;-1)$.

b) z' is pure imaginary.

$\text{Re}(z')=0$ and $\text{Im}(z')\neq 0$.

$x^2-2x+y^2+y=0$ and $x^2+(y+1)^2\neq 0$

$x^2-2x+1+y^2+y+\frac{1}{4}=\frac{5}{4}$ $\quad x\neq \quad y\neq -1$

$(x-1)^2+(y+\frac{1}{2})^2=\frac{\sqrt{5}}{4}$ and $-x+2y+2\neq 0$

The set of points M is the circle of center $I(1;-\frac{1}{2})$ and radius $R=\frac{\sqrt{5}}{2}$ excluding the points $(2;0)$ and $(0;-1)$. 1.5

III.- 1) $\int \frac{x-9}{\sqrt{x}-3}\,dx = \int \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(\sqrt{x}-3)}\,dx$

$= \int(\sqrt{x}+3)\,dx = \frac{2}{3}x\sqrt{x}+3x+C$ 2

2) $\int \frac{2x^3-9x^2+12x-2}{(x-2)^2}\,dx = \int\left(2x-1+\frac{2}{(x-2)^2}\right)dx$

$= x^2-x-\frac{2}{x-2}+C$ 2

$$\begin{array}{r|l} 2x^3-9x^2+12x-2 & x^2-4x+4 \\ \hline -2x^3+8x^2-8x & 2x-1 \\ \hline -x^2+4x-2 & \\ +x^2-4x+4 & \\ \hline 2 & \end{array}$$

1.5

3) $\int \frac{x+1}{\sqrt{3x-1}}\,dx$

let $u = \sqrt{3x-1}$

$du = \frac{3}{2\sqrt{3x-1}}\,dx$

$\frac{2}{3}du = \frac{1}{\sqrt{3x-1}}\,dx$ 0.5

$= \frac{2}{3}\int \frac{u^2+4}{3}\,du$

$= \frac{2}{9}\left(\frac{u^3}{3} + 4u\right) + c$ 0.5

$= \frac{2}{9}\left(\frac{(3x-1)\sqrt{3x-1}}{3} + 4\sqrt{3x-1}\right) + c$

$u = \sqrt{3x-1}$

$u^2 = 3x-1$

$x = \frac{u^2+1}{3}$ 0.5

IV,- 1) $S = -2 + 1 - \frac{1}{2} + \dots + \frac{1}{2^{16}}$ is a geometric sequence

$u_1 = -2 \qquad r = -\frac{1}{2}$ 0.5

$u_n = u_1 r^{n-1}$

$-2\left(-\frac{1}{2}\right)^{n-1} = \frac{1}{2^{16}}$

$\left(-\frac{1}{2}\right)^{n-1} = \left(-\frac{1}{2}\right)^{17}$ 0.5

$n-1 = 17 \qquad n = 18$

$S_{18} = u_1 \cdot \frac{1-(-\frac{1}{2})^{18}}{1+\frac{1}{2}}$

$= -2\left(\frac{2}{3}\right)\left[1-\left(-\frac{1}{2}\right)^{18}\right] = -\frac{4}{3}\left(\frac{262143}{262144}\right)$ 1

≈ -1.333

2) $u_5^2 = u_4 \cdot u_6 \qquad u_5^2 = 4096 \qquad u_5 = \pm 64$ 1

$u_5 = u_3 r^2 \qquad 64 = 1024\, r^2 \qquad r^2 = \frac{1}{16} \qquad r = \pm\frac{1}{4}$ 0.5

If $u_3 = 1024$ then $u_5 = 64$

$u_3 = u_1 r^2$

$u_1 = \frac{u_3}{r2} = 1024 \div \frac{1}{16} = 16384$ 0.5