This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 04

12FEB2024

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	11SE
Teacher :	Mr. George El Haber
Duration :	2 hours

Calculators are ALLOWED
Draw the graphs on a grid paper not on a graph paper!

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I. Consider the function $f(x) = \frac{x^2+3x+3}{x+1}$, and denote by (C) its representative curve in an orthonormal system $(O, \vec{\imath}, \vec{\jmath})$. **[25 pts.]**

1) Find the domain of definition of $f(x)$.
2) Find the limits at the boundaries of the domain of definition and deduce the asymptotes.
3) Show that $f(x)$ can be written in the form $f(x) = x + 2 + \frac{1}{x+1}$
4) Show that the line (D) with equation $y = x + 2$ is an asymptote to (C).
5) Study the relative position of (C) with respect to (D).
6) Show that $f'(x) = \frac{x(x+2)}{(x+1)^2}$.
7) Draw the table of variations.
8) Find the particular points.
9) Draw the graph.
10) Show that $I(-1; 1)$ is a center of symmetry for (C).

II. Consider the function $f(x) = \frac{x^2}{x^2+x-2}$, and denote by (C) its representative curve in an orthonormal system $(O, \vec{\imath}, \vec{\jmath})$. **[25 pts.]**

1) Find the domain of definition of $f(x)$.
2) Find the limits at the boundaries of the domain of definition and deduce the asymptotes.
3) Find the derivative of $f(x)$.
4) Draw the table of variations.
5) Find the particular points.
6) Draw the graph.

III. In a school of 200 students, 82 students are in the band, 94 students are on sport teams, and 30 students participate in both activities. **[10 pts.]**

a) Represent the above information on a Venn diagram.

b) How many students are involved in either band or sports?

IV. A jar contains 4 red marbles, 6 green marbles, and 10 white marbles. A first marble is drawn, its color is noted, then put back in the jar, then a second marble is drawn, its color is noted, then put back in the jar. What is the probability that the marbles have same color? **[10 pts.]**

----------George El Haber----------

SE04 Maths IISE Answer key

I. 1) Domain of definition: $]-\infty;-1[\cup]-1;+\infty[$ (1)

2) $\lim\limits_{x\to-\infty}\dfrac{x^2+3x+3}{x+1}=\lim\limits_{x\to-\infty}\dfrac{x^2}{x}=\lim\limits_{x\to-\infty}x=-\infty$ (1)

$\lim\limits_{x\to+\infty}\dfrac{x^2+3x+3}{x+1}=\lim\limits_{x\to+\infty}\dfrac{x^2}{x}=\lim\limits_{x\to+\infty}x=+\infty$ (1)

$\lim\limits_{x\to-1^-}\dfrac{x^2+3x+3}{x+1}=\dfrac{(-1)^2+3(-1)+3}{-1^-+1}=\dfrac{1}{0^-}=-\infty$ (1)

$\lim\limits_{x\to-1^+}\dfrac{x^2+3x+3}{x+1}=\dfrac{(-1)^2+3(-1)+3}{-1^++1}=\dfrac{1}{0^+}=+\infty$ (1)

$x=-1$ V.A (1)

3) $x+2+\dfrac{1}{x+1}=\dfrac{(x+2)(x+1)+1}{x+1}=\dfrac{x^2+3x+3}{x+1}=f(x)$ Verified. (2)

4) $\lim\limits_{x\to-\infty}[f(x)-y]=\lim\limits_{x\to-\infty}\left(x+2+\dfrac{1}{x+1}-x-2\right)=\lim\limits_{x\to-\infty}\dfrac{1}{x+1}=0^-$ (1)

$\lim\limits_{x\to+\infty}[f(x)-y]=\lim\limits_{x\to+\infty}\left(x+2+\dfrac{1}{x+1}-x-2\right)=\lim\limits_{x\to+\infty}\dfrac{1}{x+1}=0^+$ (1)

Then, the line (d) is an asymptote to (c).

5) $\lim\limits_{x\to-\infty}[f(x)-y]=0^-$ The curve (c) is below the O.A. (1)

$\lim\limits_{x\to+\infty}[f(x)-y]=0^+$ The curve (c) is above the O.A. (1)

6) $f'(x) = \frac{u'v - v'u}{v^2} = \frac{(2x+3)(x+1) - (1)(x^2+3x+3)}{(x+1)^2} = \frac{2x^2+2x+3x+3-x^2-3x-3}{(x+1)^2}$

$= \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$ Verified. (2) $x=0$ $x=-2$

7)

x	$-\infty$		-2		-1		0		$+\infty$
$f'(x)$		+	0	−	‖	−	0	+	
$f(x)$	$-\infty$	↗	-1	↘	$-\infty$ ‖ $+\infty$	↘	3	↗	$+\infty$

(2)

8) Particular Points:

x-intercept: $\varnothing$ (1)

y-intercept: $(0;3)$ (1)

Extrema: $(-2;-1)$ $(0;3)$ (1)

Table of values:

x	-1	0	1
y	1	2	3

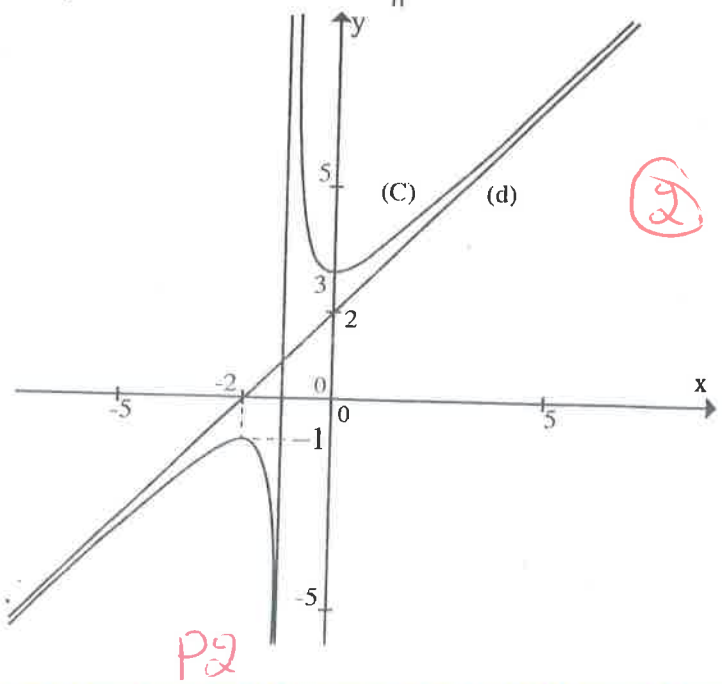
(1)

Extra points:

x	-5	-4	-1	2	4	5
y	-3.2	-2.3	3.5	4.3	6.2	7.1

(1)

9)



(2)

10) $f(2a-x)+f(x)$

$= f(-2-x)+f(x)$

$= \dfrac{(-2-x)^2+3(-2-x)+3}{(-2-x)+1} + \dfrac{x^2+3x+3}{x+1}$

$= \dfrac{4+4x+x^2-3-3x}{-(1+x)} + \dfrac{x^2+3x+3}{x+1}$

$= \dfrac{1+x+x^2}{-(1+x)} + \dfrac{x^2+3x+3}{x+1}$

$= \dfrac{-(1+x+x^2)+x^2+3x+3}{1+x}$ (2)

$= \dfrac{-1-x-x^2+x^2+3x+3}{1+x}$

$= \dfrac{2+2x}{1+x}$

$= \dfrac{2(1+x)}{(1+x)} = 2 = 2b$ Verified

so $I(-1;1)$ is a center of symmetry for (C).

II - 1) Domain of definition: $]-\infty;-2[\cup]-2;1[\cup]1;+\infty[$ (1)

2) $\lim\limits_{x\to-\infty}\dfrac{x^2}{x^2+x-2}=\lim\limits_{x\to-\infty}\dfrac{x^2}{x^2}=1$ (1)

$\lim\limits_{x\to+\infty}\dfrac{x^2}{x^2+x-2}=\lim\limits_{x\to+\infty}\dfrac{x^2}{x^2}=1$ (1)

$\Big\}\ y=1$ H.A. (1)

$\lim\limits_{x\to-2^-}\dfrac{x^2}{x^2+x-2}=\dfrac{(-2)^2}{(-2^-)^2+(-2^-)-2}=\dfrac{4}{4^--2^--2}=\dfrac{4}{0^-}=+\infty$ (1)

$\lim\limits_{x\to-2^+}\dfrac{x^2}{x^2+x-2}=\dfrac{(-2)^2}{(-2^+)^2+(-2^+)-2}=\dfrac{4}{4^+-2^+-2}=\dfrac{4}{0^+}=-\infty$ (1)

then $x=-2$ V.A. (1)

$\lim\limits_{x\to1^-}\dfrac{x^2}{x^2+x-2}=\dfrac{(1)^2}{1^-+1^-2}=\dfrac{1}{0}=-\infty$ (1)

$\lim\limits_{x\to1^+}\dfrac{x^2}{x^2+x-2}=\dfrac{1}{(1^+)+1^+-2}=\dfrac{1}{0^+}=+\infty$ (1)

$\Big\}\ x=1$ V.A. (1)

3) $f'(x)=\dfrac{u'v-v'u}{v^2}=\dfrac{2x(x^2+x-2)-(2x+1)(x^2)}{(x^2+x-2)^2}$ (3)

$=\dfrac{2x^3+2x^2-4x-(2x^3+x^2)}{(x^2+x-2)^2}$

$=\dfrac{x^2-4x}{(x^2+x-2)^2}=0$

$x=0$ $\quad x=4$

4)

x	$-\infty$		-2		0		1		4		$+\infty$
$f'(x)$		$+$	$\|\|$	$+$	0	$-$	$\|\|$	$-$	0	$+$	
$f(x)$	1	↗	$+\infty$ $\|\|$ $-\infty$	↗	0	↘	$+\infty$ $\|\|$ $-\infty$	↘	$\frac{8}{9}$	↗	1

3

5) Particular Points:

x- intercept: (0;0) 1

y-intercept: (0;0) 1

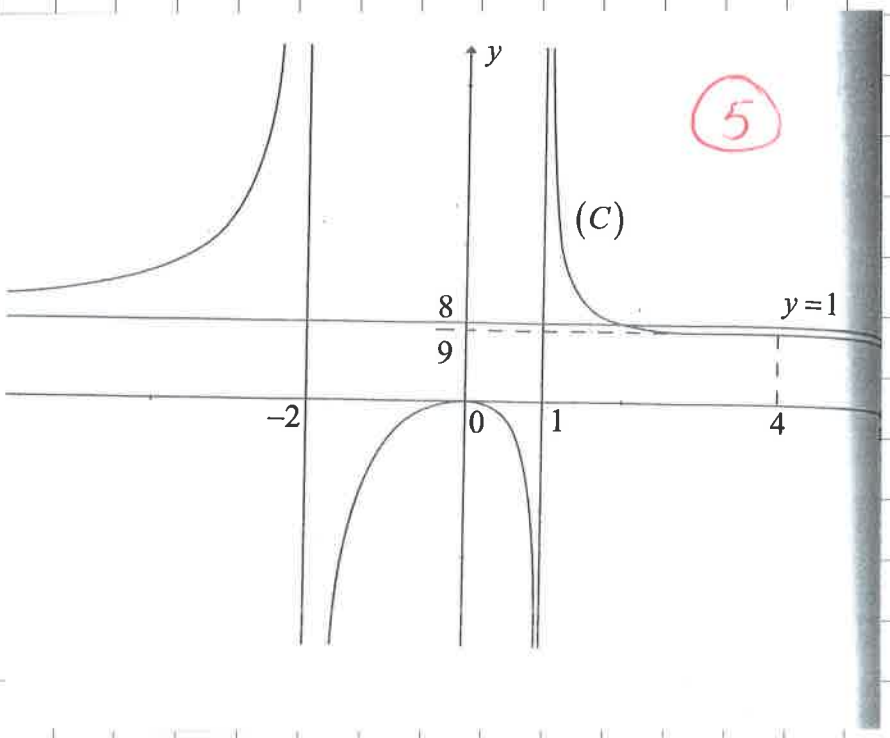
Extrema : (0;0) $(4;\frac{8}{9})$ 1

Extra points:

x	-3	2	3	5
y	2.2	1	0.9	0.8

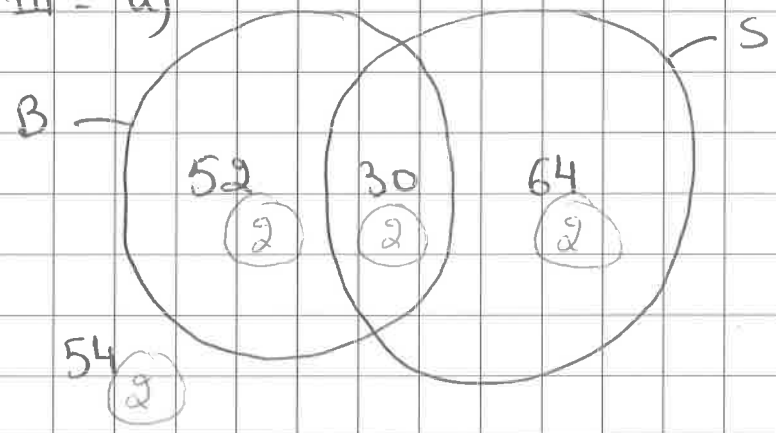
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6)



5

III - a)



E 200 Students in school
B 82 students in Band
S 94 students in Sports

B ∩ S 30 students in Both Activities

b) $\text{Card}(B \cup S) = \text{Card}(B) + \text{Card}(S) - \text{Card}(B \cap S)$
$= 82 + 94 - 30$
$= 146$ (2)

IV - $P(\text{Same color}) = P(2 \text{ red}) + P(2 \text{ green}) + P(2 \text{ white})$
$= \left(\frac{4}{20} \times \frac{4}{20}\right) + \left(\frac{6}{20} \times \frac{6}{20}\right) + \left(\frac{10}{20} \times \frac{10}{20}\right)$ (1)
$= \frac{16 + 36 + 100}{400}$ (1)
$= \frac{152}{100} = \frac{19}{50}$ (3)