This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School
for Boys and Girls — Loueizeh - Baabda

Stage Exam 01

17FEB2025

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	11SE
Teacher :	Mr. George Mouhayar
Duration :	1 hour
You are to answer the questions to this exam on : Squared paper	

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I. Using the definition of the derivative only, calculate the derivative of the following function at the point $a = 1$. (2.5 points)

$$f(x) = x^2 - 5x + 3$$

II. Calculate the derivative of each of the following functions. (7 points)

a) $f(x) = x^7 - 3x^3 + \sqrt{x} - \pi^2$

b) $f(x) = 2(-2x + 3)^4$

c) $f(x) = \frac{3x^2 - 1}{2x + 1}$

d) $f(x) = (2x^2 + 1)(x + 5)$

III. Calculate the limit of the following functions. (4 points)

a) $\lim\limits_{x \to +\infty} \frac{2x^2 - 10x}{x^3 + 1}$

b) $\lim\limits_{x \to 3} \frac{\sqrt{x+1} - 2}{x - 3}$

IV. Consider the function $f(x) = \frac{x + 6}{-x + 3}$ (6.5 points)

a) Find the domain of definition of $f(x)$.

b) Calculate the limit of $f(x)$ at $+\infty$ and $-\infty$ and deduce the horizontal asymptote.

c) Determine the vertical asymptote for the curve of $f(x)$.

d) Calculate the derivative of $f(x)$.

e) Find the equation of the tangent to the graph of $f(x)$ at the point A (0; 2).

Solution For SE01

Class 1

11 SE Maths. 2024 2025

I. $f(x) = x^2 - 5x + 3 \qquad a = 1$

$f(a) = (1)^2 - 5(1) + 3$
$= -1$

$f(a+h) = (h+1)^2 - 5(h+1) + 3$
$= h^2 + 2h + 1 - 5h - 5 + 3$
$= h^2 - 3h - 1$ (0.5 pt.)

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ (0.5)

$f'(1) = \lim_{h \to 0} \frac{h^2 - 3h - 1 - (-1)}{h}$

$= \lim_{h \to 0} \frac{h^2 - 3h - 1 + 1}{h}$ (0.5)

$= \lim_{h \to 0} \frac{h(h-3)}{h}$ (0.5)

$= \lim_{h \to 0} h - 3$

$f'(1) = -3$ (0.5)

II. a) $f(x) = x^7 - 3x^3 + \sqrt{x} - \pi^2$

$f'(x) = 7x^6 - 9x^2 + \frac{1}{2\sqrt{x}}$ (1 pt.)

b) $f(x) = 2(-2x+3)^4$
$f'(x) = 8(-2x+3)^3(-2)$
$= -16(-2x+3)^3$ (1.5 pt.)

c) $f(x) = \dfrac{3x^2 - 1}{2x+1}$

$u(x) = 3x^2 - 1$ $\quad u'(x) = 6x$

$v(x) = 2x+1$ $\quad v'(x) = 2$

$$f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$$

$$= \frac{6x(2x+1) - (3x^2-1)(2)}{(2x+1)^2}$$ (1.5)

$$= \frac{12x^2 + 6x - 6x^2 + 2}{(2x+1)^2}$$ (0.5)

$$f'(x) = \frac{6x^2 + 6x + 2}{(2x+1)^2}$$ (0.5)

d) $f(x) = (2x^2+1)(x+5)$

$u(x) = 2x^2+1$ $\quad v(x) = x+5$

$u'(x) = 4x$ $\quad v'(x) = 1$

$$f'(x) = u'(x)v(x) + u(x)v'(x)$$

$$= 4x(x+5) + (2x^2+1)(1)$$ (1.5)

$$= 4x^2 + 20x + 2x^2 + 1$$

$$f'(x) = 6x^2 + 20x + 1$$ (0.5)

III.

a) $\lim_{x \to +\infty} \frac{2x^2 - 10x}{x^3 + 1}$

$\lim_{x \to +\infty} \frac{2x^2}{x^3} = \lim_{x \to +\infty} \frac{2}{x}$

$= 0^+$ (1)

b) $\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x - 3} = \frac{0}{0}$ (0.5) Indeterminate form

$\lim_{x \to 3} \frac{\sqrt{x+1} - 2}{x - 3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$ (0.5)

$\lim_{x \to 3} \frac{(\sqrt{x+1})^2 - 2^2}{(x-3)(\sqrt{x+1} + 2)}$ (0.5)

$\lim_{x \to 3} \frac{x + 1 - 4}{(x-3)(\sqrt{x+1} + 2)}$ (0.5)

$\lim_{x \to 3} \frac{x - 3}{(x-3)(\sqrt{x+1} + 2)}$ (0.5)

$\lim_{x \to 3} \frac{1}{\sqrt{x+1} + 2} = \frac{1}{\sqrt{3+1} + 2}$

$= \frac{1}{4}$ (0.5)

IV. $f(x) = \dfrac{x+6}{-x+3}$

a) $-x+3 \neq 0$

$x \neq 3$ (0.5)

$D_f : x \in]-\infty; 3[\cup]3; +\infty[$

b) $\lim\limits_{x \to +\infty} \dfrac{x+6}{-x+3}$ $\lim\limits_{x \to -\infty} \dfrac{x+6}{-x+3}$

$\lim\limits_{x \to +\infty} \dfrac{x}{-x} = -1$ (0.5) $\lim\limits_{x \to -\infty} \dfrac{x}{-x} = -1$

$\Rightarrow y = -1$ (0.5) is a horizontal asymptote

c) $\lim\limits_{x \to 3^-} \dfrac{x+6}{-x+3} = \lim\limits_{x \to 3^-} \dfrac{9}{0^+} = +\infty$ (1)

$\lim\limits_{x \to 3^+} \dfrac{x+6}{-x+3} = \lim\limits_{x \to 3^+} \dfrac{9}{0^-} = -\infty$

$\Rightarrow x = 3$ is a vertical asymptote (0.5)

d) $f'(x) = \dfrac{(1)(-x+3) - (x+6)(-1)}{(-x+3)^2}$ (1.5)

$= \dfrac{-x+3+x+6}{(-x+3)^2}$

$f'(x) = \dfrac{9}{(-x+3)^2}$ (0.5)

A(0; 2)

e) $f'(0) = \dfrac{9}{(0+3)^2} = 1$ (slope) (0.5)

$y - y_A = f'(0)(x - x_A)$ (0.5)

$y - 2 = 1(x - 0)$

$y = x + 2$ (0.5)