This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School

for Boys and Girls Loueizeh - Baabda

Stage Exam 04

27MAR2023

Do NOT write in this box. Your MARK will be placed here.

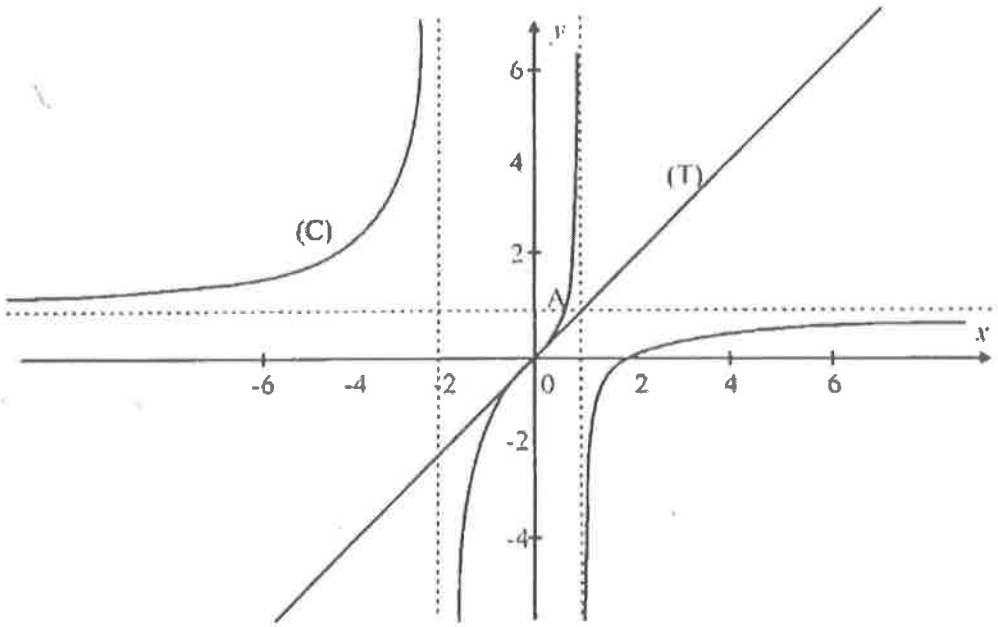
Subject :	Maths
Class :	11S
Teacher :	Taline Hawatian
Duration :	1 hour

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I.- (6 pts)

The following curve (C) is the graphic representation of a function f defined over $R - \{-2; 1\}$ and (T) is the tangent to (C) at O. **The equation of (T) is $y = x$.**



Using the graph

1) Write the equations of the asymptotes to (C).
2) Determine: $f(0), f(2), f'(0)$.
3) Solve the equation $f(x) = 0$.
4) Solve the inequality $f(x) \leq 0$.
5) Discuss, according to the values of m, the number and the sign of solutions of the equation $f(x) = m$.

II.- (5 pts)

Solve one of the following equations, where x belongs to $[0; 2\pi]$.

1) $cos5x - cosx = \sqrt{2}sin3x$
2) $cosx - \sqrt{3}sinx = \sqrt{2}$

III.- (5 pts)

Consider $E = cos2x - 2tanx + 2$

1) Write E in terms of $tanx$. Use $cos2x = \frac{1-tan^2x}{1+tan^2x}$
2) Solve $E = 0$.

IV.- (4 pts) Show that: $\frac{sin2x+cos2x+1}{sin2x+cos2x-1} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{tanx}$

SE04/20222023/Maths/115

I.- 1) $x=-2$ $x=1$ $y=1$ 1.5

2) $f(0)=0$ $f(2)=0$ $f'(0)=1=\text{slope}(T)$ 1.5

3) $f(x)=0$ for $x=0$ and $x=2$ 0.5

4) $f(x)\leq 0$ for $x\in]-2;0]\cup]1;2]$ 1

5) $f(x)=m$ has

for $m<0$, 1 negative and 1 positive solution

for $m=0$, $x=0$, $x=2$

for $0<m<1$, 2 positive solutions 1.5

for $m=1$ 1 positive solution

for $m>1$ 1 negative and 1 positive solution

II.- 1) $\cos 5x-\cos x=\sqrt{2}\sin 3x$

$-2\sin\left(\frac{5x+x}{2}\right)\sin\left(\frac{5x-x}{2}\right)=\sqrt{2}\sin 3x$ 1

$-2\sin 3x\sin 2x-\sqrt{2}\sin 3x=0$

$-\sin 3x(2\sin 2x+\sqrt{2})=0$ 1

$\sin 3x=0=\sin 0$

$3x=0+2k\pi$

$x=\frac{2k\pi}{3}$ 1

$3x=\pi+2k\pi$

$x=\frac{(2k+1)\pi}{3}$

$2\sin 2x+\sqrt{2}=0$

$\sin 2x=-\frac{\sqrt{2}}{2}=\sin\left(-\frac{\pi}{4}\right)$

$2x=-\frac{\pi}{4}+2k\pi$

$x=-\frac{\pi}{8}+k\pi$

$2x=\pi+\frac{\pi}{4}+2k\pi$ 1

$x=\frac{5\pi}{8}+\frac{2k\pi}{2}$

$x\in\left\{0,\frac{2\pi}{3},\frac{4\pi}{3},2\pi,\frac{\pi}{3},\pi,\frac{5\pi}{3},\frac{7\pi}{8},\frac{15\pi}{8},\frac{5\pi}{8},\frac{13\pi}{8}\right\}$ 1

2) $\cos x - \sqrt{3}\sin x = \sqrt{2}$

$\cos x - \tan\frac{\pi}{3}\sin x = \sqrt{2}$

$\left(\cos x - \frac{\sin \pi/3}{\cos \pi/3}\sin x = \sqrt{2}\right)\left(\cos\frac{\pi}{3}\right)$ 1

$\cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x = \sqrt{2}\cos\frac{\pi}{3}$ 1.5

$\cos\left(\frac{\pi}{3} + x\right) = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$

$\frac{\pi}{3} + x = \frac{\pi}{4} + 2k\pi$ 1

$x = \frac{\pi}{4} - \frac{\pi}{3} + 2k\pi$

$x = -\frac{\pi}{12} + 2k\pi$

$\frac{\pi}{3} + x = -\frac{\pi}{4} + 2k\pi$ 1

$x = -\frac{\pi}{4} - \frac{\pi}{3} + 2k\pi$

$x = \frac{-7\pi}{12} + 2k\pi$

$x \in \left\{\frac{17\pi}{12} ; \frac{23\pi}{12}\right\}$ 0.5

III- $E = \cos 2x - 2\tan x + 2$

1) $E = \frac{1-\tan^2 x}{1+\tan^2 x} - 2\tan x + 2$

$= \frac{1 - \tan^2 x - 2\tan x - 2\tan^3 x + 2 + 2\tan^2 x}{1 + \tan^2 x}$ 1.5

$= \frac{-2\tan^3 x + \tan^2 x - 2\tan x + 3}{1 + \tan^2 x}$ 1.5

2) $E = 0 \quad -2\tan^3 x + \tan^2 x - 2\tan x + 3 = 0 \quad 1 + \tan^2 x \neq 0$

let $t = \tan x$

$-2t^3 + t^2 - 2t + 3 = 0$ 1

$t = 1$

$\tan x = 1 = \tan\frac{\pi}{4}$ 1

$x = \frac{\pi}{4} + k\pi$.

IV.

$$\frac{\sin 2x + \cos 2x + 1}{\sin 2x + \cos 2x - 1}$$

$$= \frac{2\sin x\cos x + 2\cos^2 x - 1 + 1}{2\sin x\cos x + 1 - 2\sin^2 x - 1}$$ 1

$$= \frac{2\cos x\,(\sin x + \cos x)}{2\sin x\,(\cos x - \sin x)}$$ 1

$$= \frac{1}{\tan x}\left(\frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}\right)$$

$$= \frac{1}{\tan x}\left(\frac{\tan x + 1}{1 - \tan x}\right)$$ 1

$$= \frac{1}{\tan x}\left(\frac{\tan x + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{4}\cdot\tan x}\right) = \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan x}$$ 1