This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 01

11NOV2022

Do NOT write in this box. Your MARK will be placed here.

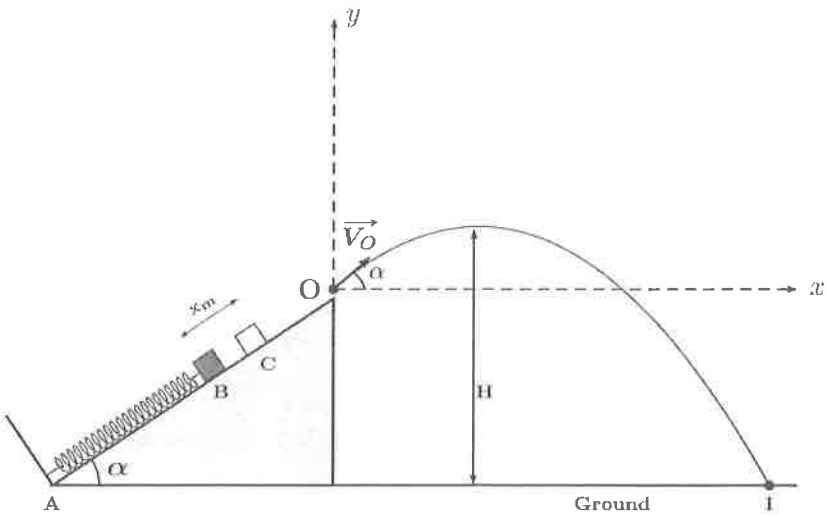
Subject :	Physics	
Class :	12E	
Teacher :	Mr. Ali Moukalled	
Duration :	1 hour	
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

EXERCISE 1 (10points)

Shooting A Bomb

To shoot a bomb, the launcher is an elastic spring of natural length $\ell_0 = 100\text{cm} = AC$ and stiffness $k = 29225\text{N/m}$, placed at the bottom of an inclined plane $AO = 4\text{m}$ making an angle $\alpha = 30°$ with the horizontal and the bomb is considered as a box (S) of mass $m = 2\text{kg}$, which contains an explosive materials. This box can slide without friction along the inclined plane. When the spring is compressed by an distance $x_m = CB = 40\text{cm}$, the box is placed next to it, then the spring is released from rest from point B. We neglect air resistance.



Take the horizontal plane passing through A is a reference level for gravitational potential energy and $g = 10\text{m/s}^2$.

A- Motion along the inclined plane

1. Determine the initial mechanical energy of the system (Spring ; Box ; Earth) at B.
2. Determine the speed V_0 of the box at O.
3. Find, in the frame of reference $(0; \vec{i}; \vec{j})$, the linear momentum vector $\vec{P_0} = P_{0x}\vec{i} + P_{0y}\vec{j}$ of the box at O

B- Flying Box

When the box leaves the inclined at O, at an origin of time $t_0 = 0$, it moves in air as a projectile motion and finally lands on the ground at I without hitting any obstacle.

1. Determine, by applying Newton's second law (the general one) the linear momentum of the box as a function of time.
2. Knowing that the velocity of the box at the maximum height (H) has no component along the y-axis, what is the value of the linear momentum of the box at the maximum height ? Deduce, from O, the time needed by the box to reach the maximum height.
3. Calculate the speed of the bomb at the maximum height.

C- Explosion of the bomb

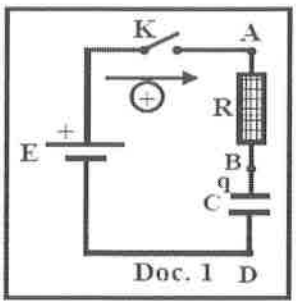
This launcher can be used to launch many bombs, one after another. One of the bomb doesn't reach the ground, because it explodes at the maximum height $(\mathbf{H = 30.8m})$ from the ground into two fragments $(\mathbf{S_1})$ and $(\mathbf{S_2})$ of masses $\mathbf{m_1 = 0.8kg}$ and $\mathbf{m_2 = 1.2kg}$ respectively. $(\mathbf{S_1})$ continues right after explosion with a velocity $\vec{V}_1 = 169.5\,\vec{i}$.
Knowing that the velocity of the bomb just before the above explosion is $\vec{V} = 41.5\,\vec{i}$.

1. The system (box) is considered to be isolated even the weight of the box is not negligible. Deduce the velocity $\vec{V}_2$ of the fragment $(\mathbf{S_2})$ just after explosion.
2. Find the mechanical energies of the system (box, Earth) just before and just after the explosion.
3. Is the mechanical energy of the system (box, Earth) conserved during explosion?

EXERCISE 2 (10points)

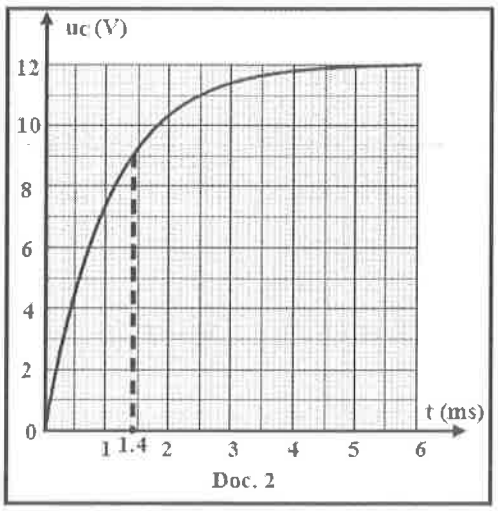
Charging a Capacitor

The aim of this exercise is to determine the capacitance C of a capacitor and the average electric power consumed by this capacitor during a certain time interval.
For this purpose, consider the series circuit of document 1 that includes a resistor of resistance $\mathbf{R = 1K\Omega}$,
an initially uncharged capacitor of capacitance C,
an ideal battery of electromotive force E, and a switch K.
The switch K is closed at $\mathbf{t_0 = 0}$.



Doc. 1

1. Name the physical phenomenon that takes place in the circuit.
2. Determine the differential equation that governs the variation of the voltage $\mathbf{u_{BD} = u_C}$ across the capacitor.
3. The solution of this differential equation is:
   $\mathbf{u_C = A + Be^{\frac{-t}{\tau}}}$.
   Determine the expressions of the constants A, B and τ in terms of E, R and C.
4. The curve of document 2 represents $\mathbf{u_C}$ as a function of time.
   (a) Referring to document 2 indicate the value of E.
   (b) Use document 2 to determine the time constant τ of the circuit.
   (c) Deduce the value of C.
   (d) Use document 2 to determine the electric energy stored in the capacitor at $\mathbf{t = 1.4ms}$.
   (e) Deduce the average electric power consumed by the capacitor between $\mathbf{t_0 = 0}$ and $\mathbf{t = 1.4ms}$.



Doc. 2

GOOD LUCK

SE01/12ME/solution

Exercise 1

A- Motion along the inclined plane

1) $M.E_B = K.E_B + G.P.E_B + E.P.E_B$

$= 0 + mgh_B + \frac{1}{2}Kx_m^2$

$= 0 + mg\,AB\sin\alpha + \frac{1}{2}Kx_m^2$

$= 2\times10\times0.6\times\sin30 + \frac{1}{2}(29225)(0.4)^2$

$= 2344\,J$

$AB = l_0 - BC$

$= 1 - 0.4$

$= 0.6m$

$\boxed{M.E_B = 2344\,J}$ (1)

2) In the absence of friction the Mechanical energy is conserved

$M.E_B = M.E_0$

$2344 = K.E_0 + G.P.E_0 + E.P.E_0$

$2344 = \frac{1}{2}mV_0^2 + mgh_0 + 0$

$2344 = \frac{1}{2}mV_0^2 + mgA0\sin\alpha$

$2344 = \frac{1}{2}(2)V_0^2 + 2\times10\times4\times\sin30$

$2344 = V_0^2 + 40$

$V_0^2 = 2304$

$V_0 = 48\,m/s$

$\boxed{V_0 = 48\,m/s}$ (1)

3) $P_{ox} = mV_{ox} = mV_o\cos\alpha$

$= 2 \times 48 \times \cos 30$

$= 83.138$ kg m/s

$P_{oy} = mV_{oy} = mV_o \sin\alpha$

$= 2 \times 48 \times \sin 30$

$= 48$ kg m/s

$$\boxed{\vec{P_o} = 83.138\,\vec{i} + 48\,\vec{j}} \quad (1)$$

B. Flying Box

1) (1) due to Newton's 2nd law

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

$$m\vec{g} = \frac{d\vec{P}}{dt} \Rightarrow d\vec{P} = m\vec{g}\,dt$$

(1) $\vec{P} = m\vec{g}\,t + C$

at $t_o = 0$, $\vec{P} = \vec{P_o}$

$83.138\,\vec{i} + 48\,\vec{j} = m\vec{g}(0) + \vec{C}$

$\Rightarrow \vec{C} = 83.138\,\vec{i} + 48\,\vec{j}$

$\vec{P} = -mgt\,\vec{j} + 83.138\,\vec{i} + 48\,\vec{j}$

(1) $$\boxed{\vec{P} = 83.138\,\vec{i} + (-20t + 48)\,\vec{j}}$$

2) $P_y = 0 \Rightarrow \vec{P}_{\text{at the maximum height}} = 83.13\,\vec{i}$

$P_y = 0 \Rightarrow -20t + 48 = 0$

$-20t = -48$

$t = \frac{48}{20} = 2.4s$

$t_{\text{to reach the maximum height}} = 2.4s$ (1)

3) $\vec{V_x} = \frac{\vec{P_x}}{m} = \frac{83.13\vec{i}}{2} = 41.56\,\vec{i}$

$V = 41.56\ m/s$ (1)

C- Explosion of the Bomb

$H = 30.8m$

$m_1 = 0.8kg \quad m_2 = 1.2kg$

$\vec{V_1} = 169.5\,\vec{i}$

1) The system is isolated, the linear momentum is conserved during the collision

$\vec{P}_{\text{system just before explosion}} = \vec{P}_{\text{system just after explosion}}$

$m\vec{V} = m_1\vec{V_1} + m_2\vec{V_2}$ (1)

$2(41.5)\vec{i} = 0.8(169.5\vec{i}) + 1.2\vec{V_2}$

$\Rightarrow \vec{V_2} = -43.83\,\vec{i}\ m/s$

2) $M.E = K.E + G.P.E$

$= \frac{1}{2} m V_x^2 + mgH$

$= \frac{1}{2}(2)(41.56)^2 + (2 \times 10 \times 30.8)$

$M.E_{\text{before explosion}} = 2343.23\ J$ (1)

$M.E_{\text{after explosion}} = K.E_1 + K.E_2 + G.P.E_1 + G.P.E_2$

$= \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + m_1 g H + m_2 g H$

$= \frac{1}{2}(0.8)(169.5)^2 + \frac{1}{2}(1.2)(43.83)^2 + (0.8 \times 10 \times 30.8) + (1.2 \times 10 \times 30.8)$

$M.E_{\text{after}} = 13260.74\ J$

3) No the M.E is NOT CONSERVED (1)

Exercise 2 — Charging a Capacitor

$R = 1K\Omega = 10^3\Omega$

1) charging of a capacitor (1)

2) Due to the law of addition of voltages

$U_g = U_R + U_c$

$E = U_R + U_c$

but $U_R = Ri$ (Ohm's Law)

$E = Ri + U_c$

but $i = C\frac{dU_c}{dt}$ (1)

$E = RC\frac{dU_c}{dt} + U_c$

$$\boxed{\frac{dU_c}{dt} + \frac{1}{RC}U_c = \frac{E}{RC}}$$ (1)

3) $U_c = A + Be^{-t/\tau}$

$\frac{dU_c}{dt} = 0 - \frac{B}{\tau}e^{-t/\tau}$

Replace U_c & $\frac{dU_c}{dt}$ in the differential Equation

$$-\frac{B}{\tau}e^{-t/\tau} + \frac{1}{RC}\left(A + Be^{-t/\tau}\right) = \frac{E}{RC}$$ (1)

$$-\frac{B}{\tau}e^{-t/\tau} + \frac{A}{RC} + \frac{B}{RC}e^{-t/\tau} = \frac{E}{RC}$$

$$\frac{A}{RC} + Be^{-t/\tau}\left(-\frac{1}{\tau} + \frac{1}{RC}\right) = \frac{E}{RC}$$ (1)

$\frac{A}{RC} = \frac{E}{RC} \Rightarrow A = E$

$Be^{-t/\tau}\left(-\frac{1}{\tau} + \frac{1}{RC}\right) = 0$ But $Be^{-t/\tau} \neq 0$

$-\frac{1}{\tau} + \frac{1}{RC} = 0$

$\frac{1}{RC} = \frac{1}{\tau} \Rightarrow \tau = RC$

$u_c = E + Be^{-t/RC}$

at $t_0 = 0$, $u_c = 0$

$0 = E + Be^0$

$0 = E + B \Rightarrow B = -E$

$$\boxed{u_c = E - Ee^{-t/RC}}$$

4) a) at $t = 5\tau$ or $t \to +\infty$ (1)

$u_c = E = 12V$

b) at $t = \tau$, $u_c = 0.63E$

$= 0.63(12)$ (1)

$= 7.56V$

Graphically $\tau = 1ms = 10^{-3}s$

c) $\tau = RC \Rightarrow C = \frac{\tau}{R} = \frac{10^{-3}}{10^3} = 10^{-6}F$

$$\boxed{C = 10^{-6}F}$$ (1)

d) $W_c = \frac{1}{2} C U_c^2 = \frac{1}{2} \times 10^{-6} \times 9^2$

$= 4.05 \times 10^{-5}$ J

$\boxed{W_c = 4.05 \times 10^{-5} \text{ J}}$ (1)

e) $P = \frac{W_c}{t} = \frac{4.05 \times 10^{-5}}{1.4 \times 10^{-3}} = 0.028$ watt

$\boxed{P = 0.028 \text{ watt}}$ (1)