This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School
for Boys and Girls Loueizeh - Baabda

# Stage Exam 01
## 06NOV2023

Do NOT write in this box. Your MARK will be placed here.

| | | |
|---|---|---|
| Subject : | Physics | |
| Class : | 11S | |
| Teacher : | Mr. Ali Moukalled | |
| Duration : | 1 hour | |
| | | |
| Your name : | | *Time taken to complete exam* |
| Your Class : | | |
| Exam Room : | | |
| How many sheets, including cover sheet : | | |

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

## Exercise1 (10 points)

A point mass (M) moves in the reference frame $(O, \vec{i}, \vec{j})$ according to the time equations :

$$\begin{cases} x = 2\sin \pi t \\ y = 2\cos \pi t \end{cases}$$

Where, $x$ and $y$ are expressed in $(m)$, and $t$ in $(sec)$.

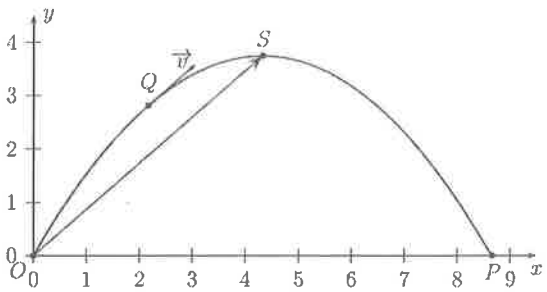
1. Show that the trajectory of $M$ is a circle whose center and radius are to be determined.
2. Write down the expression of the instantaneous position vector $\overrightarrow{OM}$.
3. Determine the expression of the instantaneous velocity vector as well as that of the instantaneous acceleration vector of M. Deduce a relation between $\vec{a}$ and $\vec{r}$.
4. Draw the trajectory of M, and then represent on it without a scale the position vector, velocity vector, and acceleration vector at $t_0 = 0$.
5. Show that the motion of M is uniform circular.
6. Show that the velocity vector is $\perp$ to the position vector, deduce the direction of the velocity vector with respect to the trajectory.
7. Show that the velocity vector $\vec{v}$ is $\perp$ to the acceleration vector $\vec{a}$. Deduce the direction of the acceleration vector.
8. Find the value of the tangential acceleration of M and deduce again the direction of the acceleration vector.

## Exercise2 (10 points)

A particle $M$ moves in a plane defined by the cartesian system $XOY$ according to the time equations :

$$\overrightarrow{OM} \begin{cases} x = 5t \\ y = -5t^2 + 5\sqrt{3}t \end{cases}$$

The adjacent figure shows the trajectory of the particle $M$ during the whole trip.




1. Determine the equation of the trajectory of the particle $M$, then specify its type.
2. Write down the expression of the instantaneous position vector $\vec{r}$ of the particle $M$.
3. Write down the expression of the instantaneous velocity vector of the particle $M$, then deduce the expression of its magnitude as a function of time.
4. Determine the instant at which the particle $M$ reaches the highest point $S$ of its trajectory.
5. Knowing that the particle $M$ was at the point $O(0,0)$ at the instant $t = 0$, determine its average velocity vector between the points $O$ and $S$.



Turn the page

6. Calculate the time taken by $M$ to reach the point $Q$ at which the instantaneous velocity vector is equal to the average velocity vector between $O$ and $S$.
7. Compare $t_Q$ and $t_S$.



GOOD LUCK

SEO1: 20232024 / Physics / 11S SOLUTION

## Exercise 1

$$\begin{cases} x = 2\sin(\pi t) \\ y = 2\cos(\pi t) \end{cases}$$

1) $\sin(\pi t) = \frac{x}{2}$ $\cos(\pi t) = \frac{y}{2}$

$\sin^2(\pi t) + \cos^2(\pi t) = 1$

$\frac{x^2}{4} + \frac{y^2}{4} = 1$ (1)

$\boxed{x^2 + y^2 = 4}$ circle of center $O(0,0)$ & radius $R = 2m$

2) $\overrightarrow{OM} = \vec{r} = x\vec{i} + y\vec{j}$

$\boxed{\vec{r} = [2\sin(\pi t)]\vec{i} + [2\cos(\pi t)]\vec{j}}$ (1)

3) * $\vec{V}_{inst} = \vec{r}' = [2\sin(\pi t)]'\vec{i} + [2\cos(\pi t)]'\vec{j}$

$\vec{V}_{inst} = [2\pi\cos(\pi t)]\vec{i} + [-2\pi\sin(\pi t)]\vec{j}$ (1)

* $\vec{a}_{inst} = \vec{V}_{inst}' = [2\pi\cos(\pi t)]'\vec{i} + [-2\pi\sin(\pi t)]'\vec{j}$

$\vec{a}_{inst} = [-2\pi^2\sin(\pi t)]\vec{i} + [-2\pi^2\cos(\pi t)]\vec{j}$ (1)

$\vec{a}_{inst} = -\pi^2[2\sin(\pi t)\vec{i} + 2\cos(\pi t)\vec{j}]$

$\boxed{\vec{a} = -\pi^2\vec{r}}$ (1)

4) $\vec{r_0} = [2\sin 0]\vec{i} + [2\cos 0]\vec{j}$

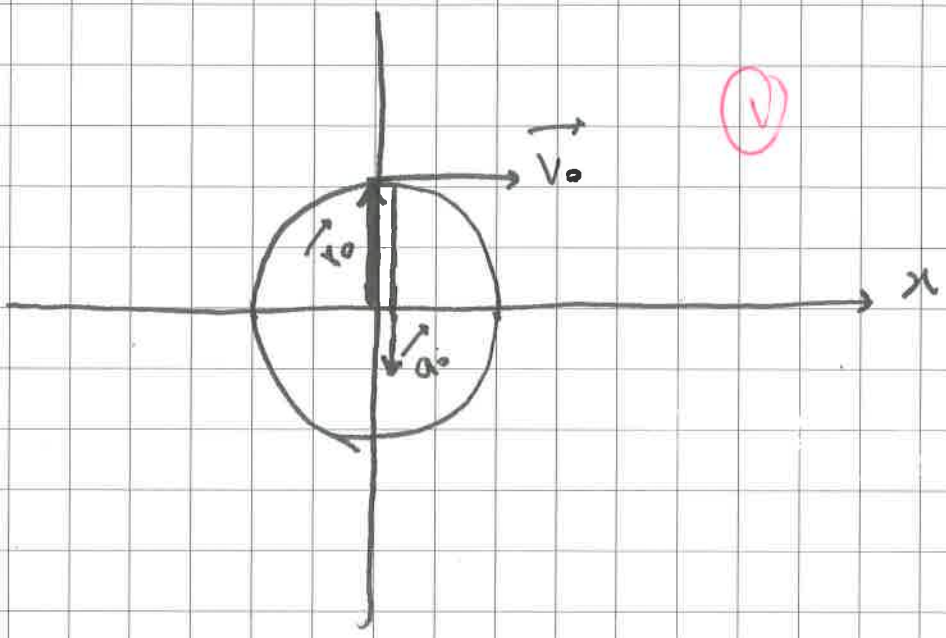
$= 0\vec{i} + 2\vec{j}$

$\vec{V_0} = 2\pi\cos 0\,\vec{i} + (-2\pi\sin 0)\vec{j}$

$= 2\pi\vec{i} + 0\vec{j}$

$\vec{a_0} = -2\pi^2\sin 0\,\vec{i} + (-2\pi^2\cos 0)\vec{j}$

$= 0\vec{i} + (-2\pi^2)\vec{j}$




(1)

5) $V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{[2\pi\cos(\pi t)]^2 + [-2\pi\sin(\pi t)]^2}$

$= \sqrt{4\pi^2[\cos^2(\pi t) + \sin^2(\pi t)]}$

$V = 2\pi$ m/s

V = constant & trajectory is a circle

$\Rightarrow$ Motion U.C.M

(1)

6) $\vec{V} \cdot \vec{r} = V_x \cdot x + V_y \cdot y$

$= [2\pi \cos(\pi t)][2\sin(\pi t)] + [-2\pi \sin(\pi t)][2\cos(\pi t)]$

$= 4\pi \sin(\pi t)\cos \pi t - 4\pi \sin(\pi t)\cos(\pi t)$

$= 0$

$\Rightarrow \vec{V} \perp \vec{r}$ (1)

7) $\vec{V} \cdot \vec{a} = V_x \cdot a_x + V_y \cdot a_y$

$= 2\pi \cos(\pi t) \cdot (-2\pi^2 \sin(\pi t)) + (-2\pi \sin(\pi t))(-2\pi^2 \cos(\pi t))$

$= 0$ (1)

$\vec{V} \perp \vec{a}$

$\vec{a} \perp$ $\vec{a}$ & $\vec{r}$ have opposite directions. (1)

8) $a_t = V' = 0 m/s^2$ (1)

$\vec{a} = \vec{a_N}$ so $\vec{a}$ is along the normal axis

## Exercise 2

1) $x = 5t$

$t = x/5$

Replace t in y

$$y = -5\left(\frac{x}{5}\right)^2 + 5\sqrt{3}\left(\frac{x}{5}\right)$$

$$\boxed{y = -\frac{x^2}{5} + \sqrt{3}x}$$ Parabola (1)

2) $\vec{r} = (5t)\vec{i} + (-5t^2 + 5\sqrt{3}t)\vec{j}$ (1)

3) $\vec{V}_{inst} = \vec{r}' = 5\vec{i} + (-10t + 5\sqrt{3})\vec{j}$

$$V = \sqrt{V_x^2 + V_y^2}$$

$$= \sqrt{5^2 + (-10t + 5\sqrt{3})^2}$$

$$= \sqrt{25 + 100t^2 - 100\sqrt{3} + 75}$$

$$\boxed{V = \sqrt{100t^2 - 100\sqrt{3}t + 100}}$$ (2)

4) $V_y = 0$

$-10t + 5\sqrt{3} = 0$ (1)

$-10t = -5\sqrt{3}$

$$\boxed{t = \sqrt{3}/2 \ s}$$

5) $V_{av} = \frac{\Delta \vec{r}}{\Delta t}$ (1)

$$= \frac{\vec{r}_{\sqrt{3}/2} - \vec{r}_0}{\sqrt{3}/2 - 0}$$

$$V_{av} = \frac{4.33\vec{i} + 3.75\vec{j}}{\sqrt{3}/2 - 0}$$ (1)

$$= 4.99\vec{i} + 4.33\vec{j}$$

6) $V_{av} = V_{inst}$ (1)

$4.99\vec{i} = 5\vec{i}$ & $4.33 = -10t + 5\sqrt{3}$

$10t = 5\sqrt{3} - 4.33$

$= 4.33$ (1)

$$\boxed{t = 0.4335}$$

7) $t_s = 2t_Q$

or $t_Q = \frac{t_s}{2}$ (1)