This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School

for Boys and Girls Loueizeh - Baabda

Stage Exam 05

27APR2026

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	12M	
Teacher :	Bechara Kordahi	
Duration :	1 hour	
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(75 points)

An urn contains nine balls :

three white, numbered from 1 to 3
three black, numbered from 1 to 3
three red , numbered from 1 to 3.

Two balls are drawn, simultaneously and at random, from this urn.

Consider the following events :

A : " the **two** drawn balls carry odd numbers " .
B : " The **two** drawn balls have the same color " .
C : " The **two** drawn balls have different colors " .
D : " The **two** drawn balls have different colors and carry two odd numbers ".

1) Calculate the following probabilities : P(A) , P(B) , P(A∩B) and P(A/B). Are the events A and B independent ?

2) a- Calculate P(C) and prove that $P(D) = \frac{1}{3}$.

b- The two drawn balls are of different colours, what is the probability that they carry two odd numbers ?

3) Let Δ be equal to the absolute value of the difference between the numbers carried by the two drawn balls.

Determine : P(Δ =0) and P(Δ =2) . Then , deduce P (Δ =1) .

Question II-(25 points)

1) Determine the following anti-derivative : $\int \frac{x}{1+x^4} dx$

2) Find the following sum in terms of n : ${}_nC_2 + {}_nC_3 + {}_nC_4 + \ldots + {}_nC_n$.

3) Given that : $f(x) = Ln(e^x - x)$ and $g(x) = \arctan(2x)$ where $x \in \Re$.
Find the derivative of $[f \circ g(x)]$ at $x = 0$.

4) If the number of diagonals of an n-sided convex polygon is 54 , find the value of n

5) Find the constant term in the expansion of $\left(2x - \frac{3}{x^3}\right)^{12}$.

(I) 3W: 1, 2, 3
3B: 1, 2, 3
3R: 1, 2, 3
} 2 balls drawn simultaneously

1) $P(A) = \frac{{}_6C_2 \cdot {}_3C_0}{{}_9C_2} = \frac{5}{12}$ 5

$P(B) = \frac{({}_3C_2) \times 3}{{}_9C_2} = \frac{1}{4}$ 5

$P(A \cap B) = P(W_1 W_3 \text{ or } B_1 B_3 \text{ or } R_1 R_3) = \frac{3}{{}_9C_2} = \frac{1}{12}$ 10

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{1}{3}$ 5

$P(A/B) = \frac{1}{3} \neq P(A) = \frac{5}{12}$ 10
NOT independent events (*)

2) (a) $P(C) = P(\bar{B}) = 1 - P(B) = \frac{3}{4}$ 5

$P(D) = P(C \cap A) = P(A \cap \bar{B}) = P(A) - P(A \cap B) = \frac{1}{3}$ 10

(b) $P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{3}}{\frac{3}{4}} = \frac{4}{9}$ 10

3) $P(\Delta = 0) = \frac{{}_3C_2 + {}_3C_2 + {}_3C_2}{{}_9C_2} = \frac{1}{4}$ 5

$P(\Delta = 2) = \frac{{}_3C_1 \cdot {}_3C_1 \cdot {}_3C_0}{{}_9C_2} = \frac{1}{4}$ 5

$P(\Delta = 1) = 1 - \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{1}{2}$ 5

II) 1) $\int \frac{x}{1+x^4}dx = \frac{1}{2}\int \frac{du}{1+u^2}$ $u=x^2$, $du=2xdx$

$\frac{du}{2} = xdx$

$= \boxed{\frac{1}{2}\arctan x^2 + C}$ 5

2) $nC_0 + nC_1 + \underbrace{nC_2 + nC_3 + \cdots + nC_n}_{\text{sum}} = \sum_{r=0}^{n} nC_r = (1+1)^n = 2^n$

$\text{Sum} = 2^n - (1+n) = \boxed{2^n - n - 1}$ 5

3) $f(x) = \ln(e^x - x)$ $g(x) = \arctan(2x)$

$I_0 = [f \circ g(x)]'_{x=0} = f'[g(x)] \cdot g'(x)_{x=0}$ $f'(x) = \frac{e^x - 1}{e^x - x}$

$I_0 = f'[g(0)] \cdot g'(0)$ $g'(x) = \frac{2}{1+4x^2}$

$I_0 = f'(0) \cdot g'(0) = 0[2] = \boxed{0}$ 5

4) $N_{\text{diagonals}} = nC_2 - n$, $54 = \frac{n!}{2!(n-2)!} - n = \frac{n(n-1)}{2} - n$

$\frac{n^2 - n - 2n}{2} = 54$, $n^2 - 3n - 108 = 0$, $\boxed{n = 12}$ 5

5) $\left(2x - \frac{3}{x^3}\right)^{12} = \left[2x + \left(\frac{-3}{x^3}\right)\right]^{12} = \sum_{r=0}^{12} {}_{12}C_r (2x)^{12-r} \left(\frac{-3}{x^3}\right)^r$

The constant term corresponds to $\frac{x^{12-r}}{x^{3r}} = x^0$

$12 - r = 3r$, $4r = 12$, $r = 3$

Then ${}_{12}C_3 (2x)^9 \left(\frac{-3}{x^3}\right)^3 = {}_{12}C_3 (2)^9 (-3)^3$

$= \boxed{-3,041,280}$ 5

2/2