This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

# Stage Exam 03

## 26JAN2024

| Do NOT write in this box. Your MARK will be placed here. |
|---|
| |

| | |
|---|---|
| Subject : | Maths |
| Class : | 10A |
| Teacher : | Mrs. Claudine Eid |
| Duration : | 1 hour |

Calculators are allowed

| | | |
|---|---|---|
| Your name : | | *Time taken to complete exam* |
| Your Class : | | |
| Exam Room : | | |
| How many sheets, including cover sheet : | | |

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

**Answer ALL questions.**

**Write your answers in the space provided**

**You must write down all the stages in your working.**

**(1)** Express $\frac{x}{x+2}+\frac{2x}{x-4}$ as a single fraction in its simplest form.

.....

**(3)**

**(Total for Question 1 is 3 marks)**

**(2)** Show that $\frac{\sqrt{12}}{\sqrt{3}+2}$

can be written in the form $a+\sqrt{b}$ where $a$ and $b$ are integers.

**(Total for Question 2 is 3 marks)**

(3) $\frac{2x+3}{x-5}+\frac{x-4}{x+5}-3$ can be written in the form $\frac{ax+b}{x^2-25}$ where $a$ and $b$ are integers.

Work out the value of $a$ and the value of $b$.

You must show all your working.

$a =$ .....

$b =$ .....

**(Total for Question 3 is 3 marks)**

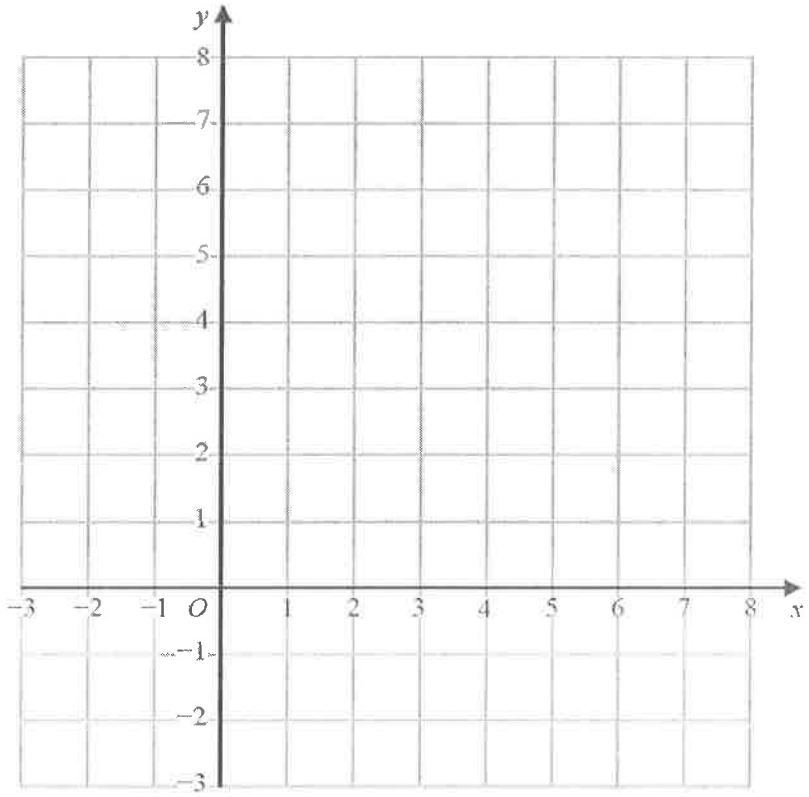
**(4)** Show that $\frac{6x^3}{(9x^2-144)} \div \frac{2x^4}{3(x-4)}$ can be written in the form $\frac{1}{x(x+r)}$ where $r$ is an integer.

**(Total for Question 4 is 3 marks)**

**(5)** On the grid show, by shading, the region that satisfies all these inequalities.

$x \geqslant 0$ $x \leqslant 2$ $y \leqslant x + 3$ $2x + 3y \geqslant 6$

Label the region **R**.




**(4)**

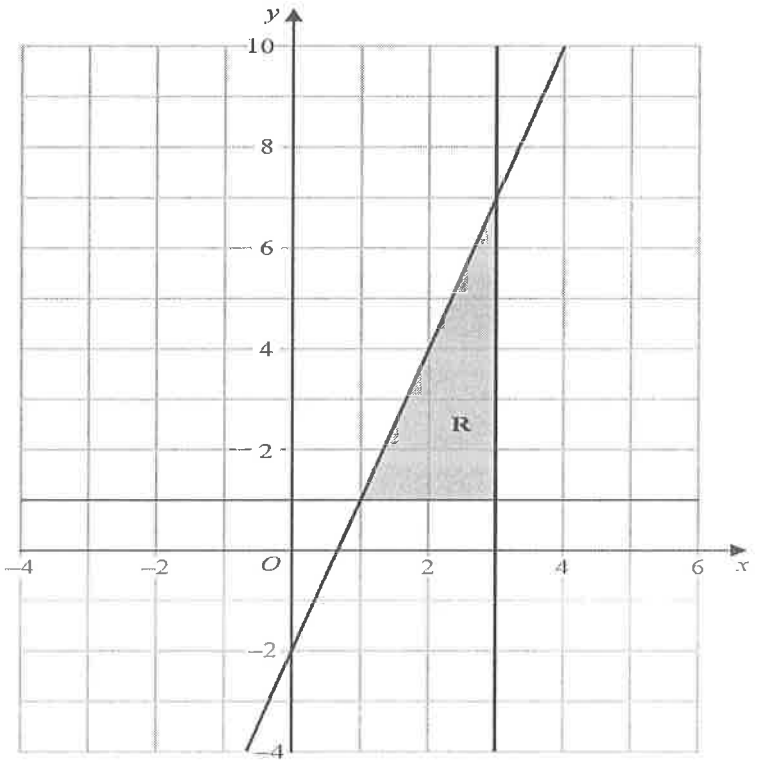
**(Total for Question 5 is 4 marks)**

**(6)** Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

**(Total for Question 6 is 4 marks)**

(7) The shaded region **R**, shown in the diagram below, is bounded by the straight line with equation $y = 3x - 2$ and by two other straight lines.

Write down the three inequalities that define region **R**.




.....

.....

.....

**(Total for Question 7 is 3 marks)**

**(8)** Find the set of values of $x$ for each of the following inequalities.

*a)* $x^2 - 49 > 0$

*b)* $-5x^2 + 31x + 72 > 0$

.....

**(Total for Question 8 is 5 marks)**

**(9)** The functions f and g are defined as

$f(x) = x^2 + 6$

$g(x) = x - 10$

(*a*) Find fg(3)

.....

**(2)**

(*b*) Solve the equation $fg(x) = f(x)$

Show clear algebraic working.

.....

**(3)**

The function h is defined as

$h(x) = \frac{2x-4}{x}$

(*c*) Find h(2)

.....

**(1)**

(*d*) Express the inverse function $h^{-1}$ in the form $h^{-1}(x) = ...$

$h^{-1}(x) =$ .....

**(3)**

**(Total for Question 9 is 9 marks)**

**(10)**

Solve the equation $\frac{5}{x+2} + \frac{3}{x^2+2x} = 2$

Show clear algebraic working.

.....

**(Total for Question 10 is 5 marks)**

**( Total for paper is 42 marks)**

**Answer ALL questions.**

**Write your answers in the space provided**

**You must write down all the stages in your working.**

**(1)** Express $\frac{x}{x+2}+\frac{2x}{x-4}$ as a single fraction in its simplest form.

$$= \frac{x(x-4)+2x(x+2)}{(x+2)(x-4)} \quad [1]$$

$$= \frac{x^2-4x+2x^2+4x}{(x+2)(x-4)} \quad [1]$$

$$= \frac{3x^2}{(x+2)(x-4)} \quad [1]$$

$\frac{3x^2}{(x+2)(x-4)}$

**(3)**

**(Total for Question 1 is 3 marks)**

**(2)** Show that $\frac{\sqrt{12}}{\sqrt{3}+2}$

can be written in the form $a+\sqrt{b}$ where $a$ and $b$ are integers.

$$\frac{\sqrt{12}}{\sqrt{3}+2} = \frac{2\sqrt{3}\,(\sqrt{3}-2)}{(\sqrt{3}+2)(\sqrt{3}-2)} = \frac{6-4\sqrt{3}}{3-4} \quad \boxed{1}\ \boxed{1}$$

$$= \frac{6-4\sqrt{3}}{-1} = 4\sqrt{3}-6 \quad \boxed{1}$$

$$\sqrt{48}-6$$

**(Total for Question 2 is 3 marks)**

**(3)** $\frac{2x+3}{x-5}+\frac{x-4}{x+5}-3$ can be written in the form $\frac{ax+b}{x^2-25}$ where $a$ and $b$ are integers.

Work out the value of $a$ and the value of $b$.

You must show all your working.

$$\frac{(2x+3)(x+5)+(x-4)(x-5)-3(x-5)(x+5)}{(x-5)(x+5)}$$ 1

$$=\frac{2x^2+13x+15+x^2-9x+20-3(x^2-25)}{x^2-25}$$ 1

$$=\frac{3x^2+4x+35-3x^2+75}{x^2-25}$$

$$=\frac{4x+110}{x^2-25}$$ 1

$a =$ 4

$b =$ 110

**(Total for Question 3 is 3 marks)**

(4) Show that $\frac{6x^3}{(9x^2-144)} \div \frac{2x^4}{3(x-4)}$ can be written in the form $\frac{1}{x(x+r)}$ where $r$ is an integer.

$$\frac{6x^3}{9(x^2-16)} \times \frac{(3)(x-4)}{(2x^4)} \quad \text{M1}$$

$$= \frac{6x^3(3)(x-4)}{9(x-4)(x+4)(2x^4)} \quad \text{M1}$$

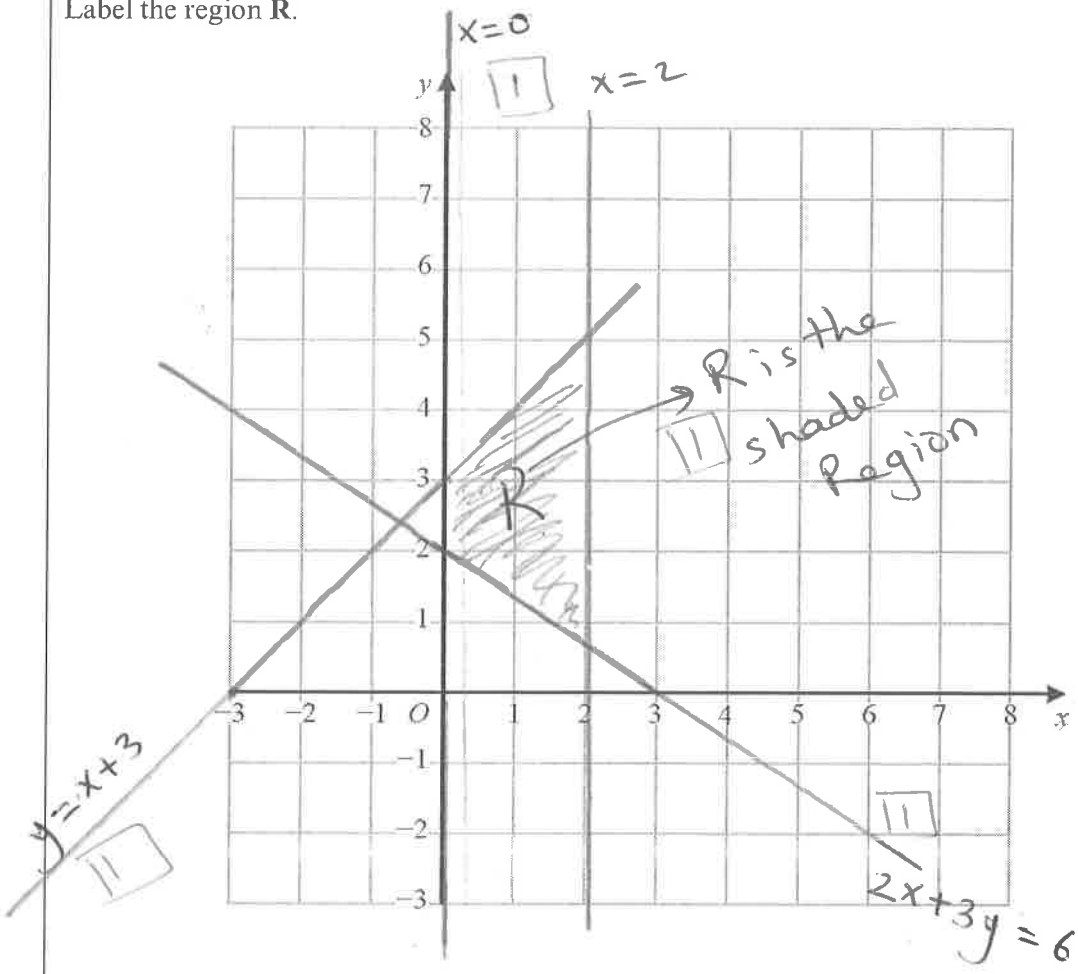
$$= \frac{1}{x(x+4)} \quad \text{M1}$$

**(Total for Question 4 is 3 marks)**

**(5)** On the grid show, by shading, the region that satisfies all these inequalities.

$x \geqslant 0$ $x \leqslant 2$ $y \leqslant x + 3$ $2x + 3y \geqslant 6$

Label the region **R**.




**(4)**

**(Total for Question 5 is 4 marks)**

**(6)** Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

$2n+1$ , $2n+3$ [1]

$(2n+3)^2 - (2n+1)^2 = 4n^2+12n+9-(4n^2+4n+1)$ [1]

$= 4n^2+12n+9-4n^2-4n-1$

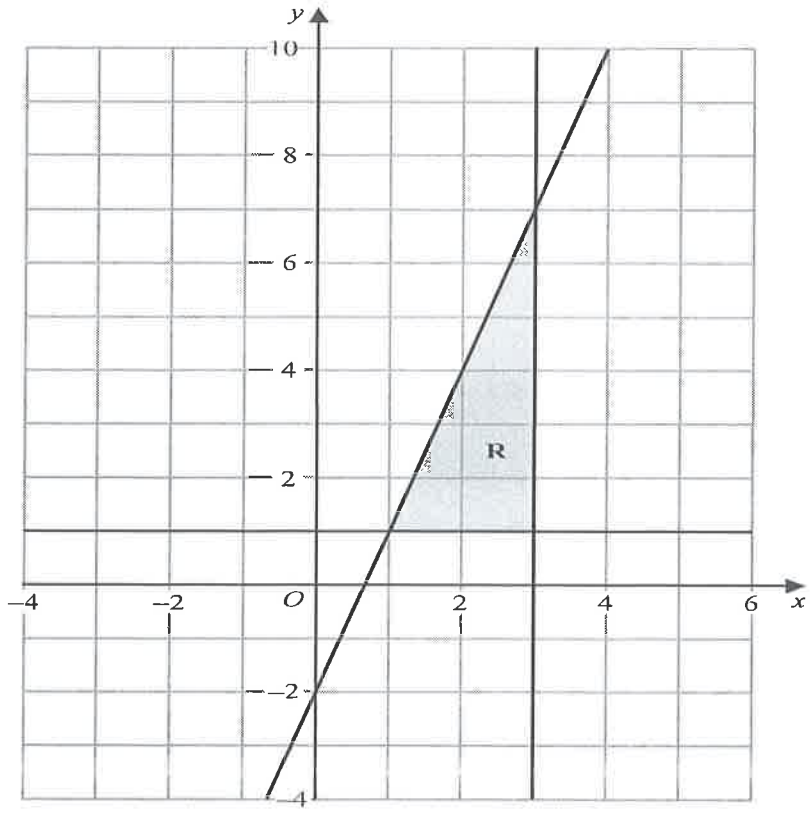
$= 8n+9-1$ [1] $= 8n+8$

$= 8(n+1)$ is a multiple of 8 [1]

**(Total for Question 6 is 4 marks)**

**(7)** The shaded region **R**, shown in the diagram below, is bounded by the straight line with equation $y = 3x - 2$ and by two other straight lines.

Write down the three inequalities that define region **R**.




$y \geqslant 1$ [1]

$y \leqslant 3x - 2$ [1]

$x \leqslant 3$ [1]

**(Total for Question 7 is 3 marks)**

**(8)** Find the set of values of $x$ for each of the following inequalities.

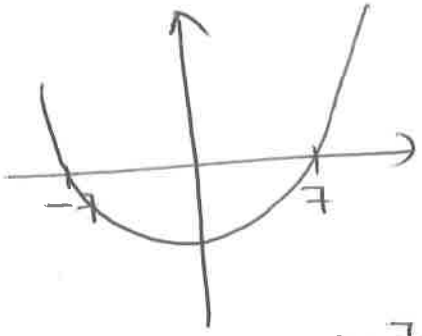
*a)* $x^2 - 49 > 0$

$y = x^2 - 49$

$y = 0$

$\Rightarrow x^2 - 49 = 0$

$x = \pm 7$ [1]




$x < -7$ or $x > 7$ [1]

*b)* $-5x^2 + 31x + 72 > 0$

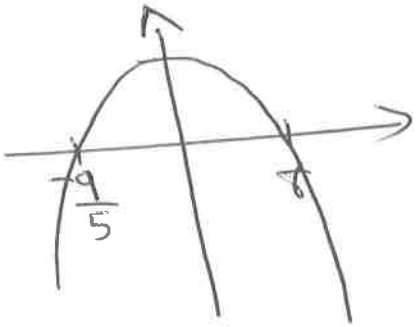
$y = -5x^2 + 31x + 72$

$y = 0$

$-5x^2 + 31x + 72 = 0$

$(-5x - 9)(x - 8) = 0$

$x = \frac{-9}{5}$ or $x = 8$ [1] [1]




$-\frac{9}{5} < x < 8$ [1]

**(Total for Question 8 is 5 marks)**

**(9)** The functions f and g are defined as

$f(x) = x^2 + 6$

$g(x) = x - 10$

(*a*) Find fg(3)

$F(3-10) = F(-7) = 49+6 = 55$ [1] [1]

.....

**(2)**

(*b*) Solve the equation $fg(x) = f(x)$

Show clear algebraic working.

$F(x-10) = f(x)$

$(x-10)^2 + 6 = x^2 + 6$ [1]

$x^2 - 20x + 100 + 6 = x^2 + 6$

$-20x = -100$ [1]

$x = 5$ [1]

.....

**(3)**

The function h is defined as

$h(x) = \frac{2x-4}{x}$

(c) Find h(2)

$h(2) = \frac{4-4}{2} = 0$ 1

.....

**(1)**

(d) Express the inverse function $h^{-1}$ in the form $h^{-1}(x) = ...$

$y = \frac{2x-4}{x}$

$x = \frac{2y-4}{y}$ 1

$xy = 2y - 4$

$xy - 2y = -4$ 1

$y(x-2) = -4$

$y = \frac{-4}{x-2}$ 1

$h^{-1}(x) = \frac{-4}{x-2}$

**(3)**

**(Total for Question 9 is 9 marks)**

**(10)**

Solve the equation $\frac{5}{x+2} + \frac{3}{x^2+2x} = 2$

Show clear algebraic working.

$$\frac{5}{x+2} + \frac{3}{x(x+2)} = 2 \quad [1]$$

$$5x + 3 = 2(x^2+2x) \quad [1]$$

$$2x^2 - x - 3 = 0 \quad [1]$$

$$x_1 = \frac{1 - \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)} = -1 \quad [1]$$

$$x_2 = \frac{1 + \sqrt{25}}{4} = \frac{6}{4} = \frac{3}{2} \quad [1]$$

**(Total for Question 10 is 5 marks)**

**( Total for paper is 42 marks)**