This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

# Stage Exam 02

## 08DEC2023

| Do NOT write in this box. Your MARK will be placed here. |
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|---|---|
| Subject : | Maths |
| Class : | 12M |
| Teacher : | Bishara Kordahi |
| Duration : | 1 hour |

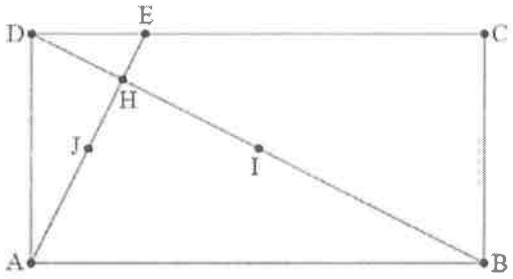
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| Your name : | | *Time taken to complete exam* |
| Your Class : | | |
| Exam Room : | | |
| How many sheets, including cover sheet : | | |

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

## Question I-(40 points)

In the adjacent figure:

- ABCD is a direct rectangle.
- AB = 8 and AD = 4.
- E is a point on [DC] such that DE = 2.
- H is the point of intersection of (BD) and (AE).
- I and J are the midpoints of [BD] and [AE] respectively.




**Part A**

Let S be the direct plane similitude that transforms B onto A and A onto D.

1) Calculate the ratio k and determine an angle $\alpha$ of S.
2) a) Show that S(D) = E.
   b) Deduce that the lines (BD) and (AE) are perpendicular.
3) Let S(C) = N. Show that EDAN is a direct rectangle.

**Part B**

Let h be the transformation defined by $h = S \circ S$.

1) a) Show that h is a dilation.
   b) Determine h(B) and h(A).
   c) Deduce the center of S.
2) Show that HI = 2HJ.

**Note :** No need to copy the drawing on the answer sheet .

## Question II –(80 points)

### Part A

Consider the function g defined on $\mathbb{R}$ as $g(x) = (-x + 2)e^{-x} + 1$.

1) Calculate g'(x).
2) Show that $g(x) > 0$ for all $x \in \mathbb{R}$.

### Part B

Consider the function f defined on $\mathbb{R}$ as $f(x) = x + 2 + (x - 1)e^{-x}$.

Denote by (C) the representative curve of f .

Let (d) be the line with equation $y = x + 2$.

1) Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-1)$.
2) a) Determine $\lim\limits_{x \to +\infty} f(x)$.
   b) Show that (d) is an asymptote to (C).
   c) Study, according to the values of x, the position of (C) with respect to (d).
3) Show that $f'(x)$ and $g(x)$ have the same sign then set up the table of variations of f.
4) Show that (C) has an inflection point W whose coordinates are to be determined.
5) a) Show that the equation $f(x) = -1$ has, on $\mathbb{R}$, a unique root $\beta$.
   b) Knowing that the equation $f(x) = 0$ has, on $\mathbb{R}$, a unique root $\alpha$, show that $\beta < \alpha$.
   c) Prove that $f\left(\frac{e^{-\beta} - 3}{e^{-\beta} + 1}\right)$ is an integer to be determined.
6) Draw (d) and (C). (Take $\alpha = -0.3$ and $\beta = -0.5$).

### Part C

Consider the function h given by $h(x) = \ln[1 + f(x)] - \ln[-f(x)]$.

1) Determine the domain of definition of h.
2) Set up the table of variations of h

**I A** $g(x) = (-x+2)e^{-x} + 1$ $D_g = \mathbb{R}$

1) $g'(x) = -e^{-x} - e^{-x}(-x+2) = e^{-x}(-1+x-2)$

$g'(x) = (x-3)e^{-x}$ 5

2) $\lim\limits_{x\to+\infty} g(x) = \lim\limits_{x\to+\infty}\left(\frac{2-x}{e^x}+1\right) = \lim\limits_{x\to+\infty}\left(\frac{-1}{e^x}+1\right) = 0+1 = 1$

No Need

$\lim\limits_{x\to-\infty} g(x) = +\infty + 1 = +\infty$

| $x$ | $-\infty$ | | $3$ | | $+\infty$ |
|---|---|---|---|---|---|
| $g'$ | | $-$ | $0$ | $+$ | |
| $g$ | $+\infty$ | $\searrow$ | | $\nearrow$ | $1$ |

5

$g_{min} = 1 - \frac{1}{e^3} \approx 0.95 > 0 \Rightarrow g(x) > 0$ for all $x \in \mathbb{R}$ $\min\left(3, 1-\frac{1}{e}\right)$

**B** (C): $f(x) = x+2+(x-1)e^{-x}$ $D_f = \mathbb{R}$ (d): $y = x+2$

1) $\lim\limits_{x\to-\infty} f(x) = -\infty - \infty = -\infty$ 3 $f(-1) = 1-2e \approx -4.4$ 2

2) (a) $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\left(x+2+\frac{x-1}{e^x}\right) = \lim\limits_{x\to+\infty}\left(x+2+\frac{1}{e^x}\right) = +\infty$ 5

(b) $\lim\limits_{x\to+\infty}\left[f(x)-(x+2)\right] = \lim\limits_{x\to+\infty}\left(\frac{x-1}{e^x}\right) = 0$ (d) asymptote to (C) as $x \to +\infty$ 5

(c) $g(x) = f(x) - (x+2) = \frac{x-1}{e^x}$ $g(x) = 0$, $x = 1$ (C) cuts (d) at $(1,3)$

$g(x) > 0$, $x > 1$ (C) above (d)

$g(x) < 0$, $x < 1$ (C) below (d) 5

3) $f'(x) = 1 + e^{-x} - e^{-x}(x-1) = 1 + 2e^{-x} - xe^{-x} = 1 + (2-x)e^{-x} = g(x)$ 5

| $x$ | $-\infty$ | | $+\infty$ |
|---|---|---|---|
| $f'$ | | $+$ | |
| $f$ | $-\infty$ | $\nearrow$ | $+\infty$ |

5

4) $f''(x) = g'(x) = (x-3)e^{-x}$ $\quad f''(x) > 0 \rightarrow x > 3$

$f''(x) < 0 \rightarrow x < 3$ 5

$f''(x) = 0, \; x = 3 \quad y = 5 + 2/e^3 \rightarrow W(3, 5+2/e^3)$

is a point of inflection

5) (a) f is continuous and strictly increases from $-\infty < 0$ to $+\infty > 0$. Thus $f(x) = -1$ has one root $\beta$. 5

(b) Also, $f(x) = 0$ has one root $\alpha$. But $f(x)$ strictly increases. Then $\beta < \alpha$. 5

(c) $f(\beta) = -1 \rightarrow \beta + 2 + (\beta - 1)e^{-\beta} = -1$

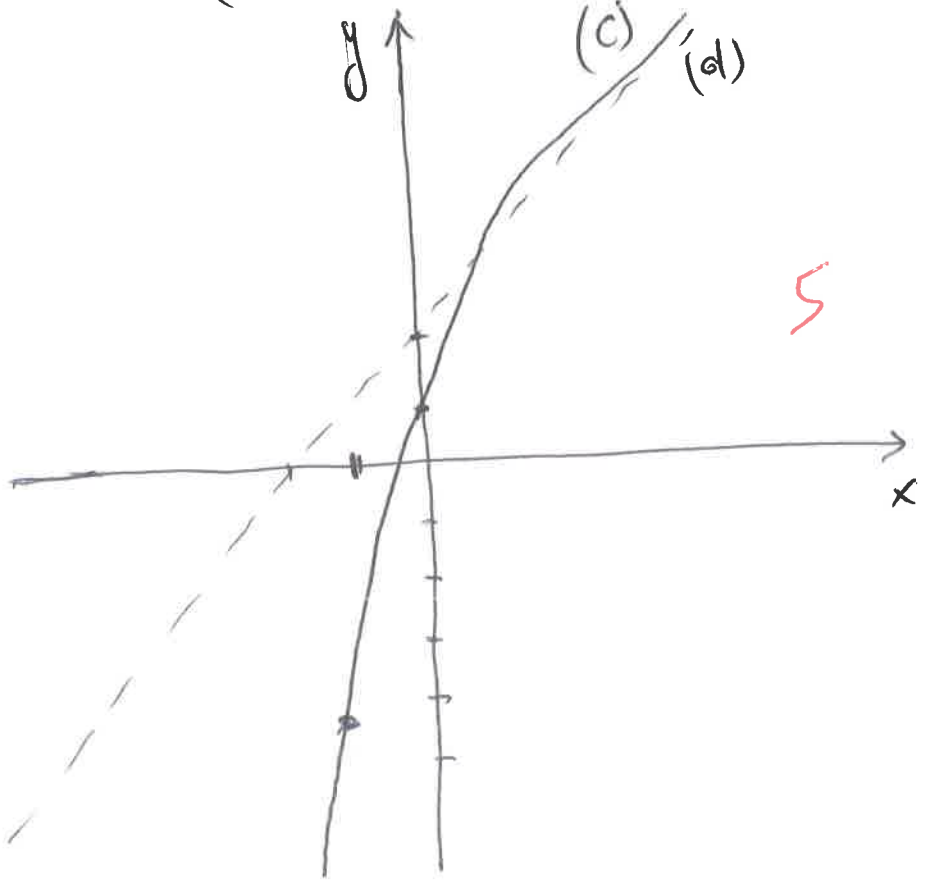
$(\beta - 1)e^{-\beta} = -\beta - 3, \quad e^{-\beta} = \frac{-\beta - 3}{\beta - 1} = \frac{3+\beta}{1-\beta}$

$$\frac{e^{-\beta} - 3}{e^{-\beta} + 1} = \frac{\frac{3+\beta}{1-\beta} - 3}{\frac{3+\beta}{1-\beta} + 1} = \frac{3 + \beta - 3 + 3\beta}{3 + \beta + 1 - \beta} = \frac{4\beta}{4} = \beta$$

10

and $f(\beta) = -1 \rightarrow$ verified

6)




5

C $h(x) = \ln[1+f(x)] - \ln[-f(x)]$

1) $\left.\begin{array}{l} 1+f(x)>0,\ f(x)>-1,\ x>\beta \\ -f(x)>0,\ f(x)<0,\ x<\alpha \end{array}\right\} \beta<x<\alpha$ 5

2) $h'(x) = \dfrac{f'(x)}{1+f(x)} - \dfrac{+f'(x)}{+f(x)} = f'(x)\left[\dfrac{f(x)<1-f(x)}{f(x)(1+f(x))}\right]$

$h'(x) = \dfrac{-f'(x)}{f(x)\cdot[1+f(x)]}$ $\quad 1+f(x)>0$, $f(x)<0$ 5

$h'(x) = \dfrac{f'(x)}{-f(x)[1+f(x)]} > 0$

| $x$ | $\beta$ | | $\alpha$ |
|---|---|---|---|
| $h'$ | | $+$ | |
| $h$ | $-\infty$ | $\nearrow$ | $+\infty$ |

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II A 1) $K = \dfrac{AD}{BA} = \dfrac{4}{8} = \dfrac{1}{2}$ 3 $\quad \alpha = (\vec{BA}, \vec{AD}) = (\vec{BA}, \vec{BC}) = \dfrac{-\pi}{2}$ 3

2) ⓐ $\dfrac{DE}{AD} = \dfrac{2}{4} = \dfrac{1}{2}$, $(\vec{AD}, \vec{AE}) = (\vec{AD}, \vec{AB}) = \dfrac{-\pi}{2}$ 3 verified

ⓑ $S([BD]) = (AE) \Rightarrow (BD, AE) = \dfrac{\pi}{2}$ 3 verified

3) $S(ABCD) = DANE$ then EDAN: direct rectangle

direct rectangles 3

S
$B \to A$
$A \to D$
$D \to E$
$C \to N$
$H \to H$
$I \to J$

B $h = SoS$

1) ⓐ $\alpha_h = 2\alpha_S = -\pi$ 2 $\to$ $h$: dilation

ⓑ $h(B) = SoS(B) = S(A) = D$ 5 $\quad (BD) \cap (AE) = \{H\}$ 5

$h(A) = SoS(A) = S(D) = E$ 5 $\quad h(H) = H \Rightarrow S(H) = H$

2) $S(H) = H$ $\quad S(I) = J$ $\quad \Rightarrow \dfrac{HJ}{HI} = \dfrac{1}{2}$

↓ mid-point of [BD] $\to$ mid-point of [AE] 5

then $HI = 2HJ$