This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 01

06NOV2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	A Level : Maths M2Pure	
Class :	12A	
Teacher :	Bishara Kordahi	
Duration :	1 hour	
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I (4 + 5 points)

(a) Prove that

$$\cot^2 x - \tan^2 x \equiv 4\cot 2x \operatorname{cosec} 2x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(b) Hence solve, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2 \theta$$

giving your answers to 2 decimal places.

Question II

(a) Express $8\sin x - 15\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R, and give the value of α, in radians, to 4 significant figures.

(3)

$$f(x) = \frac{15}{41 + 16\sin x - 30\cos x} \qquad x > 0$$

(b) Find

(i) the minimum value of $f(x)$

(ii) the smallest value of x at which this minimum value occurs.

(4)

(c) State the y coordinate of the minimum points on the curve with equation

$$y = 2f(x) - 5 \qquad x > 0$$

(1)

(d) State the smallest value of x at which a maximum point occurs for the curve with equation

$$y = -f(2x) \qquad x > 0$$

(1)

I a) $\cot^2 x - \tan^2 x = \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^4 x - \sin^4 x}{\sin^2 x \cos^2 x}$

$= \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{(\sin x \cos x)^2} = \frac{(\cos 2x)(1)}{(\frac{1}{2}\sin 2x)^2}$ 4

$= \frac{4\cos 2x}{\sin^2 2x} = \frac{4\cos 2x}{\sin 2x} \cdot \frac{1}{\sin 2x} = 4\cot 2x \operatorname{cosec} 2x$ QED

b) $4\cot 2\theta \operatorname{cosec} 2\theta = 2\tan^2\theta = \cot^2\theta - \tan^2\theta$

$3\tan^2\theta = \cot^2\theta = \frac{1}{\tan^2\theta}$, $\tan^4\theta = \frac{1}{3}$, $\tan\theta = \pm\sqrt[4]{\frac{1}{3}}$

$\theta = \pm 0.65 + k\pi \rightarrow \boxed{\theta \in \{\pm 0.65\}}$ 5

II a) $8\sin x - 15\cos x \equiv R\sin(x-\alpha) = R\sin x\cos\alpha - R\cos x\sin\alpha$

$\begin{cases} R\cos\alpha = 8 \\ R\sin\alpha = 15 \end{cases}$ $R = \sqrt{8^2+15^2} \rightarrow \boxed{R = 17}$ 1

$\cos\alpha = \frac{8}{17} \rightarrow \boxed{\alpha = 1.081}$ 1

b) $f(x) = \frac{15}{41 + 2[17\sin(x-1.081)]} = \frac{15}{41 + 34\sin(x-1.081)}$ 1

(i) $f_{min} = \frac{15}{41+34(1)} = \frac{1}{5}$ 1

(ii) $\sin(x-1.081) = 1 = \sin\frac{\pi}{2}$ $x - 1.081 = \frac{\pi}{2} + 2k\pi$

$x = 1.081 + \frac{\pi}{2} + 2k\pi$ $x_{least} = 2.652$ 2

c) $y = 2f(x) - 5 \rightarrow y_{min} = 2(\frac{1}{5}) - 5 = -\frac{23}{5}$ 1

d) $x_{least} = \frac{2.652}{2} = 1.326$ 1