This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School

for Boys and Girls Loueizeh - Baabda

Stage Exam 01

11NOV2022

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	11S
Teacher :	Taline Hawatian
Duration :	1 hour

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I.- (3 points)

Solve the following equation in x: $x = \sqrt{2x^2 - 7} + 3$.

II.- (12 points)

Consider the equation (E) in x: $(m-1)x^2 - 2(3m+2)x + 2m - 4 = 0$, where

m is a real parameter, x_1 and x_2 are its roots when they exist.

1) Determine m so that the equation has a single root.
2) Show that the discriminant $\Delta' = 7m^2 + 18m$.
3) Discuss and study, according to the values of m, the existence and sign of the roots of (E).
4) Calculate m so that x_1 and x_2 are reciprocals.
5) Show that the roots of the equation cannot be opposites.
6) Calculate m so that $\frac{(m-1)x^2-2(3m+2)x+2m-4}{-x^2+2x-2}$ is strictly positive for all x in R.

III.- (5 points)

Consider the equation (E) in x: $2mx^2 - 3x - m + 1 = 0$, where m is a real parameter,

x_1 and x_2 are its roots when they exist.

1) Show that the equation (E) has roots in R for all m.
2) Calculate, in terms of m, the value of the expression
   A= ${x_1}^2 + {x_2}^2 - (x_1 + x_2)$.

SE01/20222023/Maths/11S

I.- $x = \sqrt{2x^2-7} + 3$

$(x-3)^2 = (\sqrt{2x^2-7})^2$ 1

$x^2 - 6x + 9 = 2x^2 - 7$

$x^2 + 6x - 16 = 0$

$(x-2)(x+8) = 0$ 1

$x = 2 \qquad x = -8$

Check: $\sqrt{2(4)-7} + 3 = 1 + 3 = 4 \neq 2$ $x = 2$ rejected

$\sqrt{2(64)-7} + 3 = 11 + 3 = 14 \neq -8$ $x = -8$ rejected 1

II.- $(E): (m-1)x^2 - 2(3m+2)x + 2m - 4 = 0$

1) $a = 0$ $m - 1 = 0$ $m = 1$ $\quad -10x + 2 - 4 = 0$ 1

$-10x = 2 \quad x = \frac{-2}{10} = \frac{-1}{5}$.

2) $\Delta' = (3m+2)^2 - (m-1)(2m-4)$

$= 9m^2 + 12m + 4 - 2m^2 + 4m + 2m - 4$ 1

$= 7m^2 + 18m = m(7m+18)$

3) $\Delta' = m(7m+18) \qquad m_1 = 0 \qquad m_2 = -\frac{18}{7}$

$P = \frac{c}{a} = \frac{2m-4}{m-1} \qquad m \neq 1 \qquad m = 2$ 1

$S = -\frac{b}{a} = \frac{2(3m+2)}{m-1} \qquad m \neq 1 \qquad m = -\frac{2}{3}$

m	$-\infty$		$-18/7$		$-2/3$		0		1		2		$+\infty$
Δ'	+		0	−		−	0	+		+		+	
P	+			+		+		+	‖	−	0	+	
S	+			+	0	−		−	‖	+		+	
	$0 < x_1 < x_2$		No real roots			No real roots		$x_1 < x_2 < 0$	‖			$0 < x_1 < x_2$	

$x_1 < 0 < x_2$

$|x_1| < x_2$

3

$\Delta = 0 \quad m = 0 \quad x = \frac{-b'}{a} = \frac{3m+2}{m-1} = -2$ ✓1

$m = -\frac{18}{7} \quad x = \frac{3m+2}{m-1} = \frac{8}{5}$

2 double roots

$P = 0 \quad m = 2 \quad x_1 = 0 \quad x_2 = S = \frac{2(3m+2)}{m-1} = 16$ ✓1

$S = 0 \quad m = -\frac{2}{3}$ No real roots

4) $x_1 = \frac{1}{x_2} \quad x_1 x_2 = 1 \quad P = 1$

$\frac{2m-4}{m-1} = 1$ $\quad \Delta' \geq 0$ for $m \leq -\frac{18}{7}$ or $m \geq 0$

$2m - 4 = m - 1$ ✓1

$m = 3$ accepted

5) $x_1 = -x_2$ $\quad \Delta' \geq 0$ for $m \leq -\frac{18}{7}$ or $m \geq 0$

$x_1 + x_2 = 0 \quad S = 0$

$\frac{2(3m+2)}{m-1} = 0$ ✓1

$m = -\frac{2}{3}$ rejected

for $m = -\frac{2}{3}$

$\Delta' < 0$ No real roots

6) $-x^2 + 2x - 2$

$\Delta' = 1 - 2 = -1$

$\Delta' < 0$ No real roots

$a = -1 < 0$ ✓1

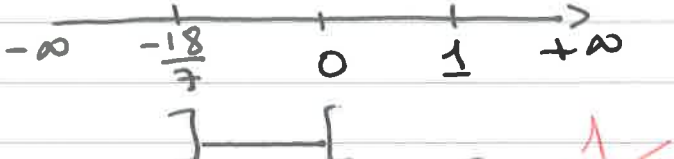
$-x^2 + 2x - 2 < 0$

for all x in $\mathbb{R}$

$(m-1)x^2 - 2(3m+2)x + 2m - 4 < 0$

$\Delta' < 0$ for $-\frac{18}{7} < m < 0$

$a < 0$ for $m < 1$





✓1

$\frac{(m-1)x^2 - 2(3m+2)x + 2m - 4}{-x^2 + 2x - 2} > 0 \quad m \in \left]-\frac{18}{7}; 0\right[$

III. $(E): 2mx^2 - 3x - m + 1 = 0$

1)

$\Delta = 9 - 4(2m)(-m+1)$

$= 9 + 8m^2 - 8m$

$= 8m^2 - 8m + 9$ ✓1

$\Delta'_m = 16 - 72 = -56 < 0$ No real roots

$a = 8 \quad a > 0$

$\Delta > 0$ for all m in $\mathbb{R}$.

(E) has 2 real roots for all x in $\mathbb{R}$. ✓1

2) $A = x_1^2 + x_2^2 - (x_1 + x_2)$

$= (x_1 + x_2)^2 - 2x_1x_2 - (x_1 + x_2)$ ✓1

$= S^2 - 2P - S$

$= \left(\frac{3}{2m}\right)^2 - \frac{2(-m+1)}{2m} - \frac{3}{2m}$ ✓1

$= \frac{9}{4m^2} + \frac{4m^2 - 4m}{4m^2} - \frac{6m}{4m^2}$

$= \frac{4m^2 - 10m + 9}{4m^2}$ ✓1