This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 02

26MAR2025

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	10B, 10C and 10E	
Teacher :	Taline Hawatian	
Duration :	1 hour	
You are to answer the questions to this exam on : Squared paper		
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I.- (7 points)

1) Solve the following equations.
   a) $(x-7)^3 = -125$ b) $(2x-1)^6 = 64$ c) $(x+10)^4 = -10^{-4}$
2) Simplify the following expression: $\frac{16^{n+3}-16^n}{6^{n+1}-6^n}$.
3) Use rational exponents to express the following: $\frac{\sqrt[3]{\sqrt{x^3}} \times \sqrt[4]{x^8}}{x\sqrt[5]{-x^4}}$ $(x > 0)$

II.- (7 points)

In a system $(O;\ \vec{\imath};\ \vec{\jmath})$, given the vectors $\vec{v} = 10\vec{\imath} - 2\vec{\jmath}$, $\overrightarrow{OA} = \vec{\imath} - 2\vec{\jmath}$, $\overrightarrow{OB} = -4\vec{\imath} - \vec{\jmath}$ and the points C $(-3; 3)$, $G(-4; 2)$ and $E(x; y)$.

1) Determine the coordinates of A and B. Express $\overrightarrow{OC}$ in terms of $\vec{\imath}$ and $\vec{\jmath}$.
2) Are the vectors $\overrightarrow{AB}$ and $\vec{v}$ collinear? Justify your answer.
3) Calculate the coordinates of E knowing that $\overrightarrow{BE} = -4\overrightarrow{AB} + \overrightarrow{BC}$.
4) Given the point M$(m+1; 2m+1)$. Calculate the components of vector $\overrightarrow{AM}$ in terms of m. Calculate m so that the points A, C, and M are collinear.
5) Calculate the coordinates of E such that G is the center of gravity of triangle ABE.
6) Calculate the coordinates of C in the system $(A;\ \vec{\imath};\ \vec{\jmath})$.

III.- (6 points)

1) Given the polynomial $P(x) = (m+10)x^3 + (m-1)x^2 + (5m-6)x + 30$. Calculate m so that -3 is a root of $P(x)$.
2) Let $P(x) = 3x^3 - 8x^2 - 41x + 30$.
   a) Show that $(x+3)$ is a factor of $P(x)$.
   b) Let $P(x) = (x+3)Q(x)$. Use long division to determine $Q(x)$.
   c) Factorize $Q(x)$. Write $P(x)$ as the product of three factors.
   d) Solve the equation $P(x) = 0$.

SE02/2024 2025/ Maths / 10BCE

I- 1) a) $(x-7)^3 = -125$ 0.5

$x - 7 = \sqrt[3]{-125}$

$x - 7 = -5$ 0.5

$x = 7 - 5 = 2$

b) $(2x-1)^6 = 64$ 0.5

$2x - 1 = 2$ $\quad$ $2x - 1 = -2$

$2x = 3$ $\quad$ $2x = 1 - 2$

$x = \frac{3}{2}$ 0.5 $\quad$ $2x = -1$ $\quad$ $x = -\frac{1}{2}$ 0.5

c) $(x+10)^4 = -10^{-4}$ impossible, no solution 1

2) $\frac{16^{n+3} - 16^n}{6^{n+1} - 6^n} = \frac{16^n(16^3 - 1)}{6^n(6-1)}$ 1 0.5

$= \left(\frac{16}{6}\right)^n \times \frac{4095}{5} = 819 \times \left(\frac{8}{3}\right)^n$ 0.5

3) $\frac{\sqrt[3]{\sqrt{x^3}} \times \sqrt[4]{x^8}}{x\sqrt[5]{-x^4}} = \frac{\sqrt[6]{x^3} \cdot x^{\frac{8}{4}}}{x \cdot (-x^{\frac{4}{5}})} = \frac{x^{\frac{1}{2}} \cdot x^2}{-x \cdot x^{4/5}}$ 0.5 0.5

$= -x^{\frac{5}{2} - \frac{9}{5}} = -x^{\frac{7}{10}}$ 0.5

II- $\vec{u} = 10\vec{i} - 2\vec{j}$, $\vec{OA} = \vec{i} - 2\vec{j}$; $\vec{OB} = -4\vec{i} - \vec{j}$,

$C(-3; 3)$, $G(-4; 2)$, $E(x; y)$.

1) $A(1; -2)$ $\quad$ $B(-4; -1)$ $\quad$ $\vec{OC} = -3\vec{i} + 3\vec{j}$ 1

2) $\vec{AB} \begin{vmatrix} x_B - x_A = -4 - 1 = -5 \\ y_B - y_A = -1 + 2 = 1 \end{vmatrix}$ $\quad$ $\vec{AB}(-5; 1)$ 0.25

$\vec{AB}(-5;1)$ $\vec{u}(10;-2)$

$\det(\vec{AB};\vec{u}) = \begin{vmatrix} -5 & 1 \\ 10 & -2 \end{vmatrix} = (-5)(-2) - 10(1)$ ✓1

$= 10 - 10 = 0.$ ✓0.25

$\det(\vec{AB},\vec{u}) = 0 \Rightarrow \vec{AB}$ and $\vec{u}$ are collinear vectors.

3) $\vec{BE} = -4\vec{AB} + \vec{BC}$

$\vec{OE} - \vec{OB} = -4(-5\vec{i} + \vec{j}) + \vec{OC} - \vec{OB}$

$\vec{OE} = 20\vec{i} - 4\vec{j} - 3\vec{i} + 3\vec{j} - \vec{OB} + \vec{OB}$ ✓1

$\vec{OE} = 17\vec{i} - \vec{j}$

$E(17;-1).$

4) $M(m+1;2m+1)$

$\vec{AM} \begin{cases} x_m - x_A = m+1-1 = m \\ y_m - y_A = 2m+1+2 = 2m+3 \end{cases}$ ✓0.5

A, C, M are collinear points.

$\Rightarrow \vec{AC}$ and $\vec{AM}$ are collinear vectors.

$\vec{AC} \begin{cases} x_C - x_A = -3-1 = -4 \\ y_C - y_A = 3+2 = 5 \end{cases}$ $\vec{AC}(-4;5)$

$\det(\vec{AC};\vec{AM}) = 0$

$\begin{vmatrix} -4 & 5 \\ m & 2m+3 \end{vmatrix} = 0$

$-4(2m+3) - 5m = 0$ ✓1

$-8m - 12 - 5m = 0$

$-13m = 12$ $\quad m = -\frac{12}{13}$

5) G is the center of gravity of $\triangle ABE$.

$x_G = \frac{x_A + x_B + x_E}{3}$ ✓0.5

$\frac{-4}{1} \times \frac{1 - 4 + x_E}{3}$ $\quad -3 + x_E = -12$ $\quad x_E = -9$

$$y_G = \frac{y_A + y_B + y_E}{3}$$

$$2 = \frac{-2 - 1 + y_E}{3}$$ 0.5

$$-3 + y_E = 6 \qquad y_E = 6 + 3 = 9 \qquad E(-9; 9)$$

6) $(A; \vec{i}; \vec{j})$

$$X_C = x_C - x_A = -3 - 1 = -4$$ 0.5

$$Y_C = y_C - y_A = 3 + 2 = 5$$ 0.5

$$C(-4; 5)$$

III, 1) $P(x) = (m+10)x^3 + (m-1)x^2 + (5m-6)x + 30$

-3 is a root of $P(x)$. $P(-3) = 0$

$$P(-3) = (m+10)(-27) + (m-1)(9) + (5m-6)(-3) + 30$$
$$= -27m - 270 + 9m - 9 - 15m + 18 + 30$$
$$= -33m - 231$$ 1

$$P(-3) = 0 \qquad -33m - 231 = 0$$
$$-33m = 231$$
$$m = \frac{231}{-33} = -7.$$

2) $P(x) = 3x^3 - 8x^2 - 41x + 30$

a) $P(-3) = 3(-27) - 8(9) - 41(-3) + 30$
$= -81 - 72 + 123 + 30 = 0.$ 0.5

-3 is a root of $P(x)$. $(x+3)$ is a factor of $P(x)$.

b)

$$\begin{array}{r|l} & 3x^2 - 17x + 10 \\ \hline x+3 & 3x^3 - 8x^2 - 41x + 30 \\ \ominus & 3x^3 \oplus 9x^2 \\ \hline & -17x^2 - 41x \\ & \oplus -17x^2 \oplus 51x \\ \hline & 10x + 30 \\ & \ominus 10x \oplus 30 \\ \hline & 0 \end{array}$$

1.5

$Q(x) = 3x^2 - 17x + 10$

c) $Q(5) = 3(25) - 17(5) + 10 = 75 - 85 + 10 = 0$.
5 is a root of $Q(x)$. 0.25

(5)

	3	-17	10
	0	15	-10
	3	-2	0

1

$Q(x) = (x-5)(3x-2)$

$P(x) = (x-5)(3x-2)(x+3)$ 0.25

d) $P(x) = 0$

$(x-5)(3x+2)(x+3) = 0$

$x - 5 = 0$ $\quad 3x - 2 = 0$ $\quad x + 3 = 0$

$x = 5$ $\quad x = \frac{2}{3}$ $\quad x = -3$

1.5