This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School

for Boys and Girls — Loueizeh - Baabda

Stage Exam 05

12MAY2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	12M	
Teacher :	Bishara Kordahi	
Duration :	2 hours	
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(20 points)-(GS 2015 session 1)

Consider the sequence (u_n) defined by : $\begin{cases} u_1 = \frac{1}{2} \\ u_{n+1} = \left(\frac{n+1}{2n}\right).u_n \end{cases}$ for $n \geq 1$.

Let (v_n) be the sequence defined as : $v_n = Ln\left(\frac{u_n}{n}\right)$.

a- Prove that the sequence (v_n) is arithmetic whose common difference is $-Ln\,2$.

b- Express v_n in terms of n .

c- Deduce that $u_n = \frac{n}{2^n}$. What is the limit of u_n ? Justify your answer .

Question II (20 points)-(2009 and 2012 session 1: parts a and b)

Each question has one correct answer . Justify your choice for each correct answer .

No	Questions	Proposed answers		
		a	b	c
1	$\int \frac{1}{\sqrt{1-x^2}}.dx$	$Arc\sin(x-1)$	$Arc\tan\left(\frac{x}{\sqrt{1-x^2}}\right)$	$Arc\cos(x+1)$
2	$n \in \mathrm{N}^*$, $U_n = \int_0^1 \frac{e^{nx}}{e^x+1}.dx$ then $U_{n+1}+U_n =$	$\frac{e^n-1}{n}$	ne^n-1	e^n-1
3	$\int_0^1 \left(\frac{x}{1+x^4}\right)dx$	$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{8}$
4	$\int_{-1}^{1} \sqrt{1-x^2}.dx$	$\frac{\pi}{2}$	$\frac{-3\pi}{2}$	$\frac{\pi}{2}+2k\pi$

Note : part c (BK exclusive) , part d exactly done in class

Question III (60 points)-(GS 2012 session1)

A- Let h be the function defined over $]0;+\infty[$ by $h(x)=\dfrac{e^x-1}{x}$.

Denote by (C) the representative curve of h in an orthonormal system $(O\ ;\vec{i},\vec{j})$.

1) a- Verify that $h'(x)=\dfrac{(x-1)e^x+1}{x^2}$.

b- Let g be the function defined over $]0;+\infty[$ by $g(x)=(x-1)e^x$.

Set up the table of variations of g and deduce that $h'(x) > 0$.

2) a- Calculate $\lim\limits_{x\to 0} h(x)$, $\lim\limits_{x\to+\infty} h(x)$ and $\lim\limits_{x\to+\infty} \dfrac{h(x)}{x}$.

b- Set up the table of variations of h .

3) a- Write an equation of (Δ), the tangent to (C) at the point with abscissa 1.

b- Draw (Δ) and (C).

B- Consider the function f defined over $]0\ ;+\infty[$ by $f(x)=h(x)+Ln(x)$ and denote by (Γ) its representative curve in the same system as (C).

1) a- Calculate $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.

b- Set up the table of variations of the function f .

2) a- Prove that the equation $f(x) = 0$ has a unique solution α and that

$$0.3 < \alpha < 0.4$$

b- Compare $h(\alpha)$ and h(1) . Deduce that : $\ell n\,\alpha > 1-e$.

3) a- Discuss, according to the values of x, the relative positions of (C) and (Γ)

b- Draw (Γ) .

4) A is a point on (C) and B is a point on (Γ) such that A and B have the same abscissa x . m is any real number such that $m > 0$. If AB = m, prove that there exist two values of x whose product is independent of m .

12M

SE 05 Maths correction

I $u_1 = 1/2$ $\quad v_n = \ln\left(\frac{u_n}{n}\right)$ $\quad n \geq 1$

$u_{n+1} = \frac{n+1}{2n} u_n$

(a) $v_{n+1} - v_n = \ln \frac{u_{n+1}}{n+1} - \ln \frac{u_n}{n} = \ln \frac{n \cdot u_{n+1}}{(n+1) u_n}$ 5

$= \ln \frac{n\left(\frac{n+1}{2n}\right) u_n}{(n+1) u_n} = \ln \frac{1}{2} = -\ln 2 = d$ verified

(b) $v_n = v_1 + (n-1) d = \ln \frac{u_1}{1} + (n-1)(-\ln 2)$ 5

$= -\ln 2 - n \ln 2 + \ln 2 \rightarrow v_n = -n \ln 2$

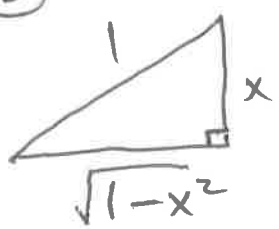
(c) $v_n = \ln\left(\frac{u_n}{n}\right) \rightarrow \frac{u_n}{n} = e^{v_n} = e^{-n \ln 2} = e^{\ln 2^{-n}}$ 5

$u_n = n\left(2^{-n}\right) \rightarrow u_n = \frac{n}{2^n}$ verified

$\lim_{n \to +\infty} u_n = \frac{\infty}{\infty} = \lim_{n \to +\infty} \frac{1}{2^n \ln 2} = \frac{1}{+\infty} = 0$ 5

II 1) $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x = \arctan \frac{x}{\sqrt{1-x^2}}$ 5

(b)



2) $u_n = \int_0^1 \frac{e^{nx}}{e^x + 1} dx \quad u_{n+1} = \int_0^1 \frac{e^{(n+1)x}}{e^x + 1}$

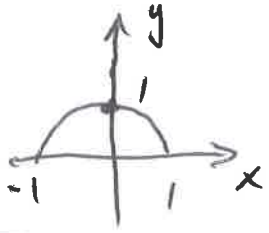
$u_{n+1} + u_n = \int_0^1 \frac{e^{nx} + e^{(n+1)x}}{e^x + 1} dx = \int_0^1 \frac{e^{nx}\left(1 + e^x\right)}{e^x + 1} dx$

$= \frac{e^{nx}}{n}\Big|_0^1 = \frac{e^n}{n} - \frac{1}{n} = \frac{e^n - 1}{n}$ (a) 5

3) $\int_0^1 \frac{x}{1+x^4}\,dx$ $\quad u = x^2,\ du = 2x\,dx,\ \frac{du}{2} = x\,dx$

$= \frac{1}{2}\int_0^1 \frac{du}{1+u^2} = \frac{1}{2}\arctan u\Big|_0^1 = \frac{1}{2}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{8}$ (c) 5

4) $\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$ (a) 5



III A (c) : $h(x) = \frac{e^x - 1}{x}$ $\quad D_h =]0, +\infty[$ (*) 2

1) (a) $h'(x) = \frac{e^x \cdot x - 1(e^x - 1)}{x^2} = \frac{(x-1)e^x + 1}{x^2}$ verified

(b) $g(x) = (x-1)e^x$ $\quad Dg =]0, +\infty[$ (*) 5

$g'(x) = 1 \cdot e^x + e^x(x-1) = xe^x$

x	0		$+\infty$
g'	▨	$+$	
g	▨	↗	$+\infty$
	-1		

2 (*) $g(x) > -1$, $(x-1)e^x > -1$

then $(x-1)e^x + 1 > 0$, $h'(x) > 0$

2) (a) $\lim_{x\to 0} h(x) = \lim_{x\to 0} \frac{e^x}{1} = 1$, $\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty} \frac{e^x}{1} = +\infty$ 2

$\lim_{x\to+\infty} \frac{h(x)}{x} = \lim_{x\to+\infty} \frac{e^x - 1}{x^2} = \lim_{x\to+\infty} \frac{e^x}{2x} = \lim_{x\to+\infty} \frac{e^x}{2} = +\infty$ 3

(b) $h'(x) > 0$

x	0		$+\infty$
h'	‖	$+$	
h	1	↗	$+\infty$

(*) 5

3) $A(1, e-1)$ $\quad h'(1) = 1$ $\quad y - (e-1) = 1(x-1) = x - 1$

$y = x - 1 + e - 1 \rightarrow \boxed{(\Delta): y = x + e - 2}$ 3

B $(\Gamma): f(x) = h(x) + \ln x \qquad D_f =]0, +\infty[$

1) (a) $\lim\limits_{x \to 0} f(x) = 1 - \infty = -\infty$, $\lim\limits_{x \to +\infty} f(x) = +\infty + \infty = +\infty$ 2

(b) $f'(x) = h'(x) + \frac{1}{x} > 0$ since $h'(x)$ and $x > 0$

x	0		$+\infty$
f'	‖	$+$	
f	‖ $-\infty$	↗	$+\infty$

(*) 5

2) (a) f is continuous and strictly increases (*) 3

from $-\infty < 0$ to $+\infty > 0 \rightarrow f(x) = 0$ unique root

$f(0,3) \times f(0,4) = -0.012 < 0$ (*) 2 verified

(b) $0,3 < \alpha < 0,4 < 1 \rightarrow \alpha < 1$ and $h(x)$ strictly increases $\rightarrow$ $h(\alpha) < h(1)$ 2

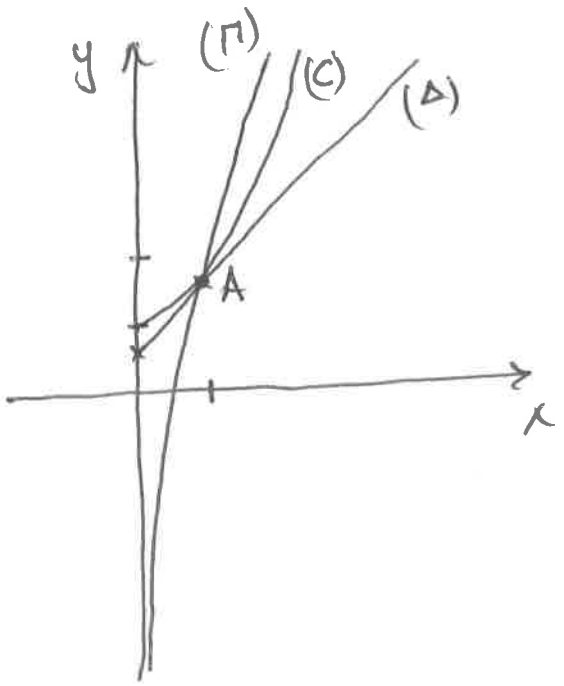
[illegible]

$f(\alpha) - \ln(\alpha) < \frac{e^1 - 1}{1}$ 3 (*) $f(\alpha) = 0$

$-\ln\alpha < e - 1 \rightarrow \ln\alpha > 1 - e$ verified

3) (a) $h(x) - f(x) = -\ln x$ $\begin{cases} x = 1 & (C) \text{ cuts } (\Gamma) \text{ at } (1, e-1) \\ 0 < x < 1 & (C) \text{ above } (\Gamma) \\ x > 1 & (C) \text{ below } (\Gamma) \end{cases}$

3 (*)



10

4) $A\left(x_1, \frac{e^x-1}{x}\right) \in (C)$ $\quad B\left(x_1, \frac{e^x-1}{x} + \ln x\right) \in (\Gamma)$

$AB = m = |\ln x| \rightarrow \ln x = \pm m, \; x = e^{\pm m}$

$x_1 = e^m \quad x_2 = e^{-m}$ $\quad \boxed{x_1 x_2 = 1}$ independent of m

4