This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 02

19DEC2022

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	11S	
Teacher :	Taline Hawatian	
Duration :	1 hour	
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I.- (8 points)

Solve the following system of inequalities and write the answer in the interval form:

$$\begin{cases} \frac{x^3-6x^2+32}{3x^2-2x-1} \geq 0 \\ \frac{5x^2+4x-27}{x^3+27} < -1 \end{cases}$$

II.- (8 points)

Consider the function f, defined on $R - \{0\}$, by $f(x) = \frac{1-x}{x}$.

1) Use the definition of the derivative of a function to calculate the derivative of f, at the point of abscissa 2.
2) Verify your answer using the formula of the derivative.
3) Write the equation of the straight line tangent to the curve of f at the point E of abscissa -2.
4) Study the existence and write the equations of the asymptotes to the curve of f.

III.- (8 points)

Calculate the following limits:

1) $\lim_{x\to 1} \frac{3x-5+2x^{-1}}{x+3-4x^{-1}}$ 2) $\lim_{x\to 9} \frac{3-\sqrt{x}}{27-\sqrt{x^3}}$

3) $\lim_{x\to -\infty} \frac{\sqrt{x^2+9}}{x+3}$ 4) $\lim_{x\to -\infty} x\left(\sqrt{x^2+1}+x\right)$

IV.- (6 points)

Calculate the derivative of each of the following functions:

1) $f(x) = \left(\frac{5x^5-3}{x^2+1}\right)^2$ 2) $f(x) = x^2 tan2x$ 3) $f(x) = \sqrt{\frac{1+sinx}{x}}$

SE02/20222023/Maths/11S

1.- $\begin{cases} \dfrac{x^3-6x^2+32}{3x^2-2x-1} \geqslant 0 \\ \dfrac{5x^2+4x-27}{x^3+27} < -1 \end{cases}$

① : $P(x) = x^3-6x^2+32$ $\qquad q(x) = 3x^2-2x-1$

$P(-2) = -8-24+32 = 0$ $\qquad = (3x+1)(x-1)$

-2 is a root of $P(x)$.

-2	1	-6	0	32
	0	-2	16	-32
	1	-8	16	0

$P(x) = (x+2)(x^2-8x+16)$

$= (x+2)(x-4)^2$ 0.5

x	$-\infty$		-2		$-1/3$		1		4		$+\infty$
$P(x)$		$-$	0	$+$		$+$		$+$	0	$+$	
$q(x)$		$+$		$+$	‖	$-$	‖	$+$		$+$	
		$-$		$+$	‖	$-$	‖	$+$		$+$	

2

$S_1 = [-2; -\frac{1}{3}[\cup]1; +\infty[$ 1

② : $\dfrac{5x^2+4x-27}{x^3+27} < -1$

$\dfrac{5x^2+4x-27+x^3+27}{x^3+27} < 0$

$\dfrac{x(x^2+5x+4)}{(x+3)(x^2-3x+9)} < 0$ 0.5 $\qquad \dfrac{x(x+4)(x+1)}{(x+3)(x^2-3x+9)} < 0$

$P(x) = x(x+4)(x+1)$

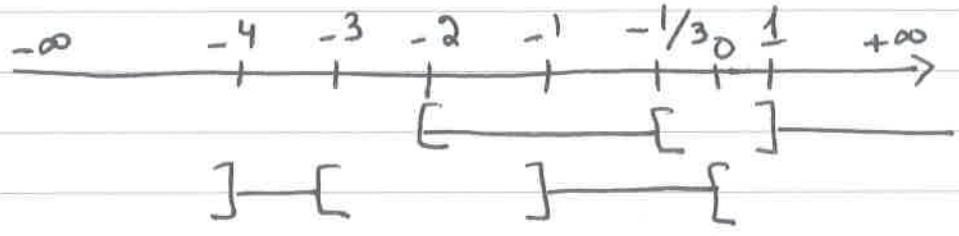
$q(x) = (x+3)(x^2-3x+9)$ $\qquad x^2-3x+9 > 0$ for all $x \in \mathbb{R}$

x	$-\infty$		-4		-3		-1		0		$+\infty$
$P(x)$		$-$	0	$+$		$+$	0	$-$	0	$+$	
$q(x)$		$-$		$-$	‖	$+$		$+$		$+$	
		$+$		$-$	‖	$+$		$-$		$+$	

2

1

$S_2 =]-4; -3[\cup]-1, 0[$



$S =]-1; -\frac{1}{3}[$

1

II.- $f(x) = \frac{1-x}{x}$

1) $f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$ $\quad f(2) = -\frac{1}{2}$ ✓0.5

$= \lim_{x\to 2} \frac{\frac{1-x}{x} + \frac{1}{2}}{x-2}$

$= \lim_{x\to 2} \frac{2-2x+x}{2x(x-2)} = \lim_{x\to 2} \frac{2-x}{2x(x-2)} = \frac{-1}{4}$ ✓1.5

2) $f'(x) = \frac{-x-(1-x)}{x^2} = \frac{-1}{x^2}$ ✓1

$f'(2) = -\frac{1}{4}$ ✓0.5

3) $f'(-2) = -\frac{1}{4}$ $\quad f(-2) = -\frac{3}{2}$ ✓0.5

$y + \frac{3}{2} = -\frac{1}{4}(x+2)$ $\quad (T): y = -\frac{1}{4}x - 2$ ✓1

4) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{1-x}{x} = \frac{1}{0^+} = +\infty$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1-x}{x} = \frac{1}{0^-} = -\infty$

$x=0$ is a vertical asymptote ✓1.5

$\lim_{x\to \pm\infty} f(x) = \lim_{x\to \pm\infty} \frac{1-x}{x} = \lim_{x\to \pm\infty} \frac{-x}{x} = -1$ $\quad y=-1$ is a horizontal asymptote ✓1.5

III.-

1) $\lim_{x\to 1} \frac{3x-5+2x^{-1}}{x+3-4x^{-1}} = \lim_{x\to 1} \frac{3x-5+\frac{2}{x}}{x+3-\frac{4}{x}}$

$= \lim_{x\to 1} \frac{3x^2-5x+2}{x^2+3x-4} = \frac{0}{0}$ Indeterminate

$= \lim_{x\to 1} \frac{(3x-2)(x-1)}{(x-1)(x+4)} = \frac{1}{5}$ ✓2

2) $\lim_{x\to 9} \frac{3-\sqrt{x}}{27-\sqrt{x^3}} = \frac{0}{0}$ Indeterminate form

$\lim_{x\to 9} \frac{3-\sqrt{x}}{27-\sqrt{x^3}} = \lim_{x\to 9} \frac{3-\sqrt{x}}{(3-\sqrt{x})(9+3\sqrt{x}+x)} = \frac{1}{27}$ ✓2

3) $\lim_{x \to -\infty} \frac{\sqrt{x^2+9}}{x+3} = \frac{+\infty}{-\infty}$ Indeterminate

$\lim_{x \to -\infty} \frac{\sqrt{x^2+9}}{x+3} = \lim_{x \to -\infty} \frac{|x|\sqrt{1+\frac{9}{x^2}}}{x\left(1+\frac{3}{x}\right)}$ 2

$= \lim_{x \to -\infty} \frac{-x\sqrt{1+\frac{9}{x^2}}}{x\left(1+\frac{3}{x}\right)} = -1.$

4) $\lim_{x \to -\infty} x\left(\sqrt{x^2+1}+x\right) = (-\infty)(+\infty-\infty)$ Indeterminate

$\lim_{x \to -\infty} x\left(\sqrt{x^2+1}+x\right).\frac{\left(\sqrt{x^2+1}-x\right)}{\sqrt{x^2+1}-x}$ 2

$= \lim_{x \to -\infty} \frac{x(x^2+1-x^2)}{\sqrt{x^2+1}-x} = \lim_{x \to -\infty} \frac{x}{x\left(-\sqrt{1+\frac{1}{x^2}}-1\right)} = \frac{-1}{2}.$

IV- 1) $f(x) = \left(\frac{5x^5-3}{x^2+1}\right)^2$

$u(x) = \frac{5x^5-3}{x^2+1}$

$u'(x) = \frac{25x^4(x^2+1)-2x(5x^5-3)}{(x^2+1)^2}$

$= \frac{15x^6+25x^4+6x}{(x^2+1)^2}$ 2.5

$f'(x) = 2\left(\frac{5x^5-3}{x^2+1}\right)\left(\frac{15x^6+25x^4+6x}{(x^2+1)^2}\right)$ 1.5

2) $f(x) = x^2\tan 2x \qquad f'(x) = 2x\tan 2x + \frac{2}{\cos^2 x}.x^2$

3) $f(x) = \sqrt{\frac{1+\sin x}{x}}$

$u(x) = \frac{1+\sin x}{x}$

$u'(x) = \frac{x\cos x - 1 - \sin x}{x^2}$

$f'(x) = \frac{\frac{x\cos x-1-\sin x}{x^2}}{2\sqrt{\frac{1+\sin x}{x}}}$ 2