This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 03
FEB2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	A Level : Maths M1Pure
Class :	11A and 12A
Teacher :	Mrs.Claudine Eid
Duration :	1 hour 20 minutes
Calculators are allowed	

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Answer ALL questions. Write your answers in the space provided

(1) Given that

$$(3pq^2)^4 \times 2p\sqrt{q^8} \equiv ap^bq^c$$

find the values of the constants a, b and c.

(3)

(Total for Question1 is 3 marks)

(2)

Find

$$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4}\,dx$$

giving each term in simplest form.

(5)

(Total for Question2 is 5 marks)

(3) $f(x) = x^4 + ax^3 - 3x^2 + bx + 5$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b.

(4)

Continue 3

Total for Question3 is 6 marks

(4) The curve C has equation $y = f(x)$, $x > 0$

Given that

- C passes through the point $P\ (8, 2)$
- $f'(x) = \frac{32}{3x^2} + 3 - 2\left(\sqrt[3]{x}\right)$

(*a*) find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(*b*) Find, in simplest form, $f(x)$.

(5)

Continue 4

(Total for Question4 is 8 marks)

(5) In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = \frac{12}{\sqrt{x}} + \frac{x}{3} - 4$
- the point $P(9, 8)$ lies on C

(a) find, in simplest form, $f(x)$

(5)

The line l is the normal to C at P

(b) Find the coordinates of the point at which l crosses the y-axis.

(4)

Continue 5

((Total for Question 5 is 9 marks)

(6) The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

Continue 6

(Total for Question6 is 11 marks)

(7) The curve C has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = \frac{2}{\sqrt{x}} + \frac{A}{x^2} + 3$, where A is a constant
- $f''(x) = 0$ when $x = 4$

(*a*) find the value of A.

(4)

Given also that

- $f(x) = 8\sqrt{3}$, when $x = 12$

(*b*) find $f(x)$, giving each term in simplest form.

(5)

Continue 7

(Total for Question7 is 9 marks)

(8) The curve C has equation $y = f(x)$

Given that

- $f(x)$ is a quadratic expression
- the maximum turning point on C has coordinates $(-2, 12)$
- C cuts the negative x-axis at -5

(a) find $f(x)$

(4)

The line l_1 has equation $y = \frac{4}{5}x$

Given that the line l_2 is perpendicular to l_1 and passes through $(-5, 0)$

(b) find an equation for l_2, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

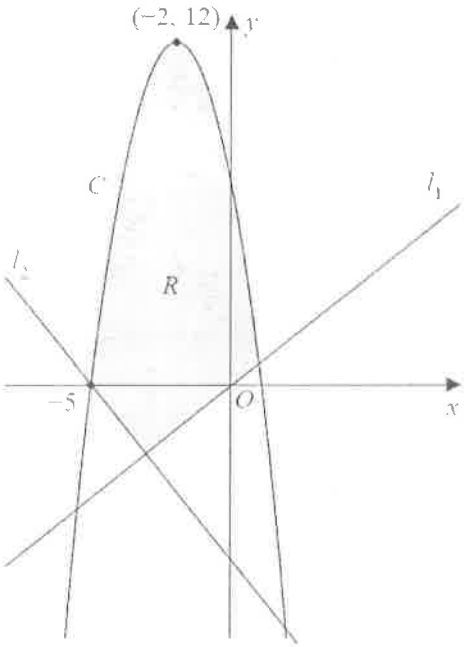


Figure 2

Figure 2 shows a sketch of the curve C and the lines l_1 and l_2

(*c*) Define the region R, shown shaded in Figure 2, using inequalities.

(2)

Continue 8

Continue8

(Total for Question8 is 9 marks)

TOTAL FOR PAPER IS 60 MARKS

Answer ALL questions. Write your answers in the space provided

(1) Given that

$$(3pq^2)^4 \times 2p\sqrt{q^8} \equiv ap^bq^c$$

find the values of the constants a, b and c.

(3)

$3^4 p^4 q^8 \times 2p \times q^4 \equiv a p^b q^c$

$3^4(2) p^5 q^{12} \equiv a p^b q^c$

$b = 5$ [1]

$c = 12$ [1]

$a = 3^4 \times 2 = 162$ [1]

(Total for Question1 is 3 marks)

(2)
Find

$$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4}\,dx$$

giving each term in simplest form.

(5)

$$\int 12x^3 + \frac{1}{6}x^{-\frac{1}{2}} - \frac{3}{2}x^{-4}\,dx$$

[1]

$$= \frac{12x^4}{4} + 2\left(\frac{1}{6}\right)x^{\frac{1}{2}} - \frac{3x^{-3}}{2(-3)} + C$$

either

$$= 3x^4 + \frac{1}{3}x^{\frac{1}{2}} + \frac{1}{2}x^{-3} + C$$

[1] [1] [1] [1]

or

$$= 3x^4 + \frac{1}{3}\sqrt{x} + \frac{1}{2x^3} + C$$

(Total for Question2 is 5 marks)

(3) $f(x) = x^4 + ax^3 - 3x^2 + bx + 5$

where a and b are constants.

When $f(x)$ is divided by $(x + 1)$, the remainder is 4

(a) Show that $a + b = -1$

(2)

When $f(x)$ is divided by $(x - 2)$, the remainder is -23

(b) Find the value of a and the value of b.

(4)

(a) $f(-1) = 4$ [1]

$1 - a - 3 - b + 5 = 4$

$-a - b = 1$

$a + b = -1$ [1]

(b) $f(2) = -23$ [1]

$16 + 8a - 12 + 2b + 5 = -23$

$8a + 2b = -32$

$4a + b = -16$ [1]

$a = -5$ [1] $\quad b = 4$ [1]

Continue 3

Total for Question3 is 6 marks

(4) The curve C has equation $y = f(x)$, $x > 0$

Given that

- C passes through the point P (8, 2)
- $f'(x) = \frac{32}{3x^2} + 3 - 2\left(\sqrt[3]{x}\right)$

(a) find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(b) Find, in simplest form, $f(x)$.

(5)

a) $f'(8) = \frac{32}{3(8)^2} + 3 - 2\sqrt[3]{8} = \frac{1}{6} + 3 - 4 = -\frac{5}{6}$ [1] [1]

$y - 2 = -\frac{5}{6}(x - 8)$ [1]

$y = -\frac{5}{6}x + \frac{20}{3} + 2 \Rightarrow y = -\frac{5x}{6} + \frac{26}{3}$ [1]

b) $\int \frac{32}{3}x^{-2} + 3 - 2x^{\frac{1}{3}}\,dx$

$= \frac{32}{3(-1)}x^{-1} + 3x - 2\left(\frac{3}{4}\right)x^{\frac{4}{3}} + c$

$= -\frac{32}{3x} + 3x - \frac{3}{2}x^{\frac{4}{3}} + c$

[1] [1] [1] [1]

Continue 4

$$2 = \frac{-32}{3(8)} + 3(8) - \frac{3}{2}(8)^{\frac{4}{3}} + C$$

$$2 = -\frac{4}{3} + 24 - 24 + C$$

$$C = 2 + \frac{4}{3} = \frac{10}{3}$$ [1]

(Total for Question4 is 8 marks)

(5) In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = \frac{12}{\sqrt{x}} + \frac{x}{3} - 4$
- the point $P(9, 8)$ lies on C

(a) find, in simplest form, $f(x)$

(5)

The line l is the normal to C at P

(b) Find the coordinates of the point at which l crosses the y-axis.

(4)

a] $\int 12x^{-\frac{1}{2}} + \frac{x}{3} - 4\,dx$

[1]

$= 12(2)x^{\frac{1}{2}} + \frac{x^2}{6} - 4x + C$

[1] [1] [1]

$= 24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x + C$

$8 = 24(3) + \frac{81}{6} - 4(9) + C$

$8 = 36 + \frac{81}{6} + C \Rightarrow C = -\frac{83}{2}$ [1]

$f(x) = 24x^{\frac{1}{2}} + \frac{x^2}{6} - 4x - \frac{83}{2}$

Continue 5

$$F'(x) = \frac{12}{\sqrt{x}} + \frac{x}{3} - 4$$

$$F'(9) = \frac{12}{3} + \frac{9}{3} - 4 = 3$$ [1]

$$\text{grad}_{\text{Normal}} = -\frac{1}{3}$$ [1]

$$y - 8 = -\frac{1}{3}(x - 9)$$

$$y = -\frac{1}{3}x + 3 + 8$$

$$y = -\frac{1}{3}x + 11$$ [1]

y axis $x = 0 \Rightarrow y = 11$ [1]

((Total for Question 5 is 9 marks)

(6) The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

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The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

a) $y = (x-2)(x^2-8x+16)$ [1]

$y = x^3 - 10x^2 + 32x - 32$ [1] [1]

$y' = \frac{dy}{dx} = 3x^2 - 20x + 32$ [1]

b] $y'_{x=6} = 3(6)^2 - 20(6) + 32 = 20$ [1]

$y_{x=6} = (6-2)^2(6-4)^2 = 16$ [1]

$y - 16 = 20(x-6)$ [1]

$y = 20x - 104$ [1]

Continue 6

l_1 and l_2 are parallel $\Leftrightarrow$ equal slopes

$3x^2 - 20x + 32 = 20$ [1]

$3x^2 - 20x + 12 = 0$ [1]

$x = 6$ $\boxed{x = \frac{2}{3}}$ [1]

found already

(Total for Question6 is 11 marks)

(7) The curve C has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = \frac{2}{\sqrt{x}} + \frac{A}{x^2} + 3$, where A is a constant
- $f''(x) = 0$ when $x = 4$

(a) find the value of A.

(4)

Given also that

- $f(x) = 8\sqrt{3}$, when $x = 12$

(b) find $f(x)$, giving each term in simplest form.

(5)

a] $f'(x) = 2x^{-\frac{1}{2}} + Ax^{-2} + 3$ [1]

$f''(x) = -x^{-\frac{3}{2}} - 2Ax^{-3}$ [1]

$f''(4) = 0 \Rightarrow 0 = -(4)^{-\frac{3}{2}} - 2A(4)^{-3}$

[1]

$0 = -\frac{1}{8} - \frac{A}{32}$

$\Rightarrow A = -4$ [1]

b] $\int 2x^{-\frac{1}{2}} - 4x^{-2} + 3\,dx$

$= 4x^{\frac{1}{2}} + 4x^{-1} + 3x + C$

[1] [1] [1] [1]

Continue 7

$$8\sqrt{3} = 4(12)^{\frac{1}{2}} + \frac{4}{12} + 3(12) + c$$

$$c = \frac{-109}{3}$$ 1

$$f(x) = 4x^{\frac{1}{2}} + 4x^{-1} + 3x - \frac{109}{3}$$

(Total for Question7 is 9 marks)

(8) The curve C has equation $y = f(x)$

Given that

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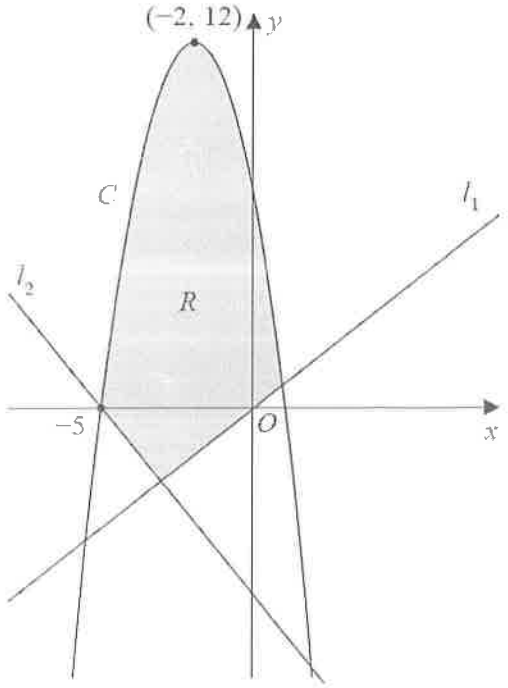


Figure 2

Figure 2 shows a sketch of the curve C and the lines l_1 and l_2

(*c*) Define the region *R*, shown shaded in Figure 2, using inequalities.

(2)

a] $y = a(x+2)^2 + 12$ [1] [i] $(-5, 0)$ [1]

$0 = a(-5+2)^2 + 12 \Rightarrow a = -\frac{4}{3}$ [1]

$f(x) = -\frac{4}{3}(x+2)^2 + 12$

Continue 8

b] $\text{grad}_{l_2} = -\frac{5}{4}$ [1] $(-5, 0)$

$y = -\frac{5}{4}(x+5)$ [1]

$y = -\frac{5x}{4} - \frac{25}{4}$ [1]

c] $y \leq -\frac{4}{3}(x+2)^2 + 12$ [1]

$y \geqslant -\frac{5x}{4} - \frac{25}{4}$ [1]

$y \geqslant \frac{4}{5}x$

Continue8

(Total for Question8 is 9 marks)

TOTAL FOR PAPER IS 60 MARKS