This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 06

19, 20, 21JUN2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	12SE
Teacher :	Nicolas Kordahi
Duration :	1 hour

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

1) (7pts)

A factory produces a certain article. The table below shows the supply y of this product, expressed in thousands of articles, in terms of the unit price x in thousands LL.

The sale price x_i in thousands LL.	2	3	5	7	10
Supply y_i in thousands of articles	5	7	8	9	11

The pattern remains valid for a price 20 000 LL

1) Determine an equation of the regression line $(D_{y/x})$.
2) Estimate the supplied number of articles for a unit price of 15 000 LL.
3) The demand is modeled by the line $(D_{z/x})$ with equation $z = -0.567x + 8.266$.
   The two lines $(D_{y/x})$ and $(D_{z/x})$ intersect at the $I(3.158\ ;\ 6.474)$.
   Give an economical interpretation of the coordinates of this point.
4) Calculate, in LL, the revenue for a unit price of 7000 LL.
5) Let $e(x)$ be the elasticity of demand in terms of the unit price x.
   a) Show that $e(x) = \frac{567x}{-567x+8266}$.
   b) Calculate $e(4)$. Give an economical interpretation of the value thus obtained.

2) (13pts)

Let f be the function defined on $[0;+\infty[$ as $f(x) = (-2x - 1)e^{-x} + 2$ and denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

Part A

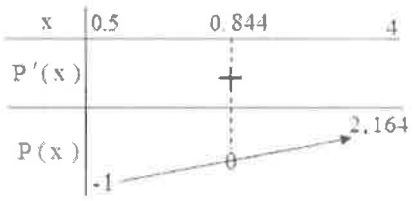
1) Determine $\lim\limits_{x \to +\infty} f(x)$. Deduce an equation of an asymptote to (C).
2) Prove that $f'(x) = (2x - 1)e^{-x}$ and set up the table of variations of f.
3) Draw (C) and its asymptote.
4) The line (d) with equation $y = \frac{x}{2}$ intersects the curve (C) at one point only with abscissa α.
   Verify that $3.5 < \alpha < 3.6$.

Part B

In what follows, suppose that $\alpha = 3.55$

A factory produces souvenirs. The total cost of production of souvenirs C_T, expressed in millions LL, is modeled as $C_T(x) = f(x)$ **only** for all x in [0.5 ; 4], where x, expressed in thousands, represents the number of souvenirs produced. $(0.5 \leq x \leq 4)$.

1) Calculate, in LL, the total cost of production of 3 000 souvenirs. In this case, deduce the average cost of production of one souvenir.
2) The adjacent table is the table of variations of the profit function P of this factory, in millions LL, over [0.5 ; 4].



   a) Study if this factory can achieve a profit equal to 3 000 000 LL.
   b) Determine the minimum number of souvenirs to be sold so that the factory realizes a profit.
   c) The profit function P is defined over [0.5 ; 4] as $P(x) = x - 2 + (2x + 1)e^{-x}$.
   Prove that the revenue function R, in millions LL, is modeled as $R(x) = x$ knowing that the whole production is sold.
   d) Calculate the number of souvenirs that should be sold so that the revenue is double the total cost.

----------NK.

1) 1) supply $D_{y/x}$: $y = 0.679x + 4.33$

2) $y = 0.679(15) + 4.33$

$= 14.515$ then 14,515 articles (2pts)

3) Demand : $Z = -0.567x + 8.266$

$I(3.158 ; 6.474)$

I is market equilibrium

3158 L.L eq. price

6474 articles eq. quantity (1pt)

4) Rev = price . (quantity demanded)

$= 7(-0.567(7) + 8.266)$ (1pt)

$= 30.079$ so Rev = 30,079 L.L

5) a) $e(x) = \frac{-x\,D'(x)}{D(x)} = \frac{-x(-0.567)}{-0.567x + 8.266}$ (1pt)

$= \frac{0.567x}{-0.567x + 8.266} = \frac{567x}{-567x + 8266}$

b) $e(4) = 0.378$ (1pt)

for price 4000 L.L, 1% ↗ in price lead to 0.378 ↘ in demand. (1pt)

2) $[0; +\infty[$ $\quad f(x) = (-2x-1)e^{-x} + 2$

A 1) $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \left(\frac{-2x-1}{e^x} + 2\right)$

$= \lim\limits_{x \to +\infty} \frac{-2}{e^x} + 2$ (1pt)

$= 2$ $\quad y = 2$ H.A

2) $f'(x) = (-2)e^{-x} + (-e^{-x})(-2x-1) + 0$

$= -2e^{-x} + 2xe^{-x} + e^{-x}$

$= e^{-x}(-2 + 2x + 1)$ (1pt)

$= e^{-x}(2x-1)$ Vérifié

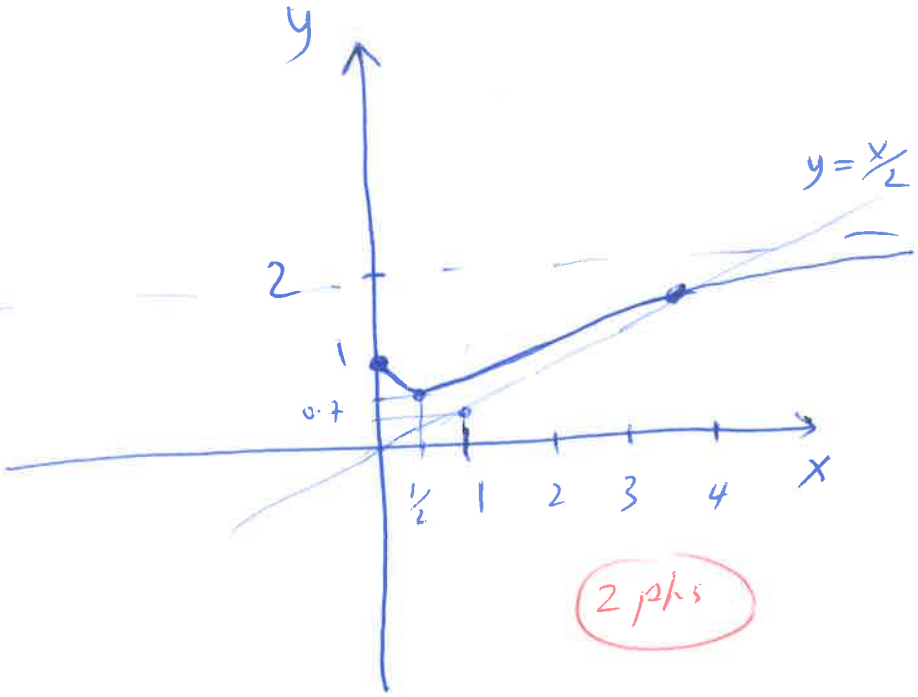
$2x - 1 = 0 \quad x = \frac{1}{2}$

x	0		$\frac{1}{2}$		$+\infty$
y'		$-$	0	$+$	
	1	↘		↗	$+2$
			Min $(\frac{1}{2}; 0.786)$		

(2pts)

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3)



2 pts

$y = \frac{x}{2}$

x	0	1
y	0	½

$f(3.5) = 1.758 > \frac{3.5}{2}$

$f(3.6) = 1.775 < \frac{3.5}{2}$

1 pt

then $3.5 < \alpha < 3.6$

B $\quad \alpha = 3.55$

$C(x) = f(x) = (-2x-1)e^{-x} + 2$

x: 1000 units

$C(x) = 1000,000$ L.L

1) $\frac{3,000}{1000} = 3$

$C(3) = (-2(3)-1)e^{-3} + 2$

$= 1.651490$ then 1,651,490 L.L

1 pt

Av. cost $= \frac{1651,490}{3000} = 550.5 \text{ L.L}$ (1pt)

2) a) according to table 2,164,000 L.L < 3,000,000 then "NO" (1pt)

b) 845 to realize a profit (from table) (1pt)

c) $p(x) = R(x) - c(x)$

$R(x) = p(x) + c(x)$

$= x - 2 + (2x+1)e^{-x} + (-2x-1)e^{-x} + 2$

$= x$ Verified (1pt)

c) $R(x) = 2\,c(x)$ (1pt)

then $f(x) = \frac{x}{2}$ then $x = 4$

then $3.55 \times 1000 = 3550$ articles

NK