This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loucizeh - Baabda

Stage Exam 06

19, 20, 21JUN2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	11SE	
Teacher :	Farhan B.	
Duration :	1 hour	
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

(1) Given the rational function: $f(x) = \frac{1}{x^2-x}$

(a) Write the domain of $f(x)$ interval form (2 marks)

(b) Find the limits of $f(x)$ at the open bounds of the domain, Deduce the asymptotes to (C) the representative curve of $f(x)$ (9 marks)

(c) Calculate the derivative $f'(x)$ (3 marks)

(d) Set the table of variations of $f(x)$ (4 marks)

(e) Graph $f(x)$ on an orthonormal system (4 marks)

(2) Simplify the following expressions $(n-1)! \times (n^2+n)$ (3 marks)

(3) The password of a new email is made of two capital letters followed by two small letters, three digits and then one symbol from the set {@, #, $, %, ^, &, *}.
{ABcd123&}
How many passwords can you make in each case?

a) Repetition is allowed, all letters and digits are accepted. (2 marks)

b) Repetition is allowed, all letters except O and all digits except 0 are accepted. (2 marks)

c) Repetition is not allowed, all letters except O and digits except 0 are accepted. (2 marks)

(4) A and B are two events such that $P(A) = \frac{5}{11}; P(A \cap B) = \frac{2}{11}; P(\bar{B}) = \frac{4}{11}$

(a) Calculate $P(B)$ and $P(A \cup B)$ (4 marks)

(b) Calculate $P(\bar{A} \cap B)$ and $P(\bar{A} \cup B)$ (4 marks)

(5) A study was conducted on 20 rivers, 3 were clean, 9 were polluted with crude oil, 12 were polluted with phosphates, some were polluted by crude oil and phosphate. Find the probability that a river is polluted with phosphates only? (8 marks)

(6) A jar contains 5 red, 7 blue and 8 yellow marbles. A first marble is drawn, its color is noted, then put back in jar, then a second is drawn, its color noted, then put back in the jar. What is the probability that the two marbles have different colors? (4 marks)

(7) Two dice are rolled, find the probability that the sum is between 4 and 9 (exclusive) (5 marks)

1) $f(x) = \dfrac{1}{x^2 - x} = \dfrac{1}{x(x-1)}$

a) Domain $]-\infty, 0[\cup]0, 1[\cup]1, +\infty[$ (2)

b) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \dfrac{1}{x^2 - x} = \lim\limits_{x \to -\infty} \dfrac{1}{x^2} = \dfrac{1}{(-\infty)^2} = 0$ (1)

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \dfrac{1}{x^2 - x} = \lim\limits_{x \to +\infty} \dfrac{1}{x^2} = \dfrac{1}{(+\infty)^2} = 0$ (1)

(1)

So $y = 0$ is a horizontal asymptote to (C) as x tends to $-\infty$ or to $+\infty$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \dfrac{1}{x(x-1)} = \dfrac{1}{0^-(0^- - 1)} = \dfrac{1}{0^+} = +\infty$ (1)

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \dfrac{1}{x(x-1)} = \dfrac{1}{0^+(0^+ - 1)} = \dfrac{1}{0^-} = -\infty$ (1)

So $x = 0$ is a vertical asymptote to (C) as x tends to 0 (1)

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \dfrac{1}{x(x-1)} = \dfrac{1}{1(1^- - 1)} = \dfrac{1}{0^+} = +\infty$ (1)

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \dfrac{1}{x(x-1)} = \dfrac{1}{1(1^+ - 1)} = \dfrac{1}{0^+} = +\infty$ (1)

So $x = 1$ is a vertical asymptote to (C) as x tends to 1 (1)

(2) (1)

c) $f'(x) = \dfrac{0(x^2-x) - 1(2x-1)}{(x^2-x)^2} = \dfrac{-2x+1}{(x^2-x)^2}$

d)

x	$-\infty$		0		$\frac{1}{2}$		1		$+\infty$
$f'(x)$		$+$	‖	$+$	0	$-$	‖	$-$	
$f(x)$	0	↗	$+\infty$ ‖ $-\infty$	↗	-4	↘	$+\infty$ ‖ $-\infty$	↘	0

(1) (1) (2)

e)



(4)

2) $(n-1)! \times (n^2+n) = (n-1)! \times n \times (n+1) = (n+1)!$ (2) (1)

3)

a) $26 \times 26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 7$ (2)

b) $25 \times 25 \times 25 \times 25 \times 9 \times 9 \times 9 \times 7$ (2)

c) $25 \times 24 \times 23 \times 22 \times 9 \times 8 \times 7 \times 7$ (2)

4) a) $P(B) = 1 - P(\bar{B}) = 1 - \frac{4}{11} = \frac{7}{11}$ (2)

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{11} + \frac{7}{11} - \frac{2}{11} = \frac{10}{11}$ (2)

b) $P(\bar{A} \cap B) = P(B) - P(A \cap B) = \frac{7}{11} - \frac{2}{11} = \frac{5}{11}$ (2)

$P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B) = \frac{6}{11} + \frac{7}{11} - \frac{5}{11} = \frac{8}{11}$ (2)

5) C : the river is polluted with crude oil $P(C) = \frac{9}{20}$

P : the river is polluted with phosphate $P(p) = \frac{12}{20}$

$P \cup C$: the river is polluted by either oil or phosphate

$P(P \cup C) = 1 - \frac{3}{20} = \frac{17}{20}$ (2)

$P \cap C$: the river is polluted by crude oil and phosphate

$P(P \cap C) = P(P) + P(C) - P(P \cup C) = \frac{12}{20} + \frac{9}{20} - \frac{17}{20} = \frac{4}{20}$ (2)

Polluted by phosphate only $P \cap \bar{C}$ (1)

$P(P \cap \bar{C}) = P(P) - P(P \cap C) = \frac{12}{20} - \frac{4}{20} = \frac{8}{20} = \frac{2}{5}$ (1) (1) (1)

6) 5 red, 7 blue, 8 yellow marbles total 20

$P(\text{Same color}) = P(rr) + P(bb) + P(yy)$ (1)

$= \frac{5}{20} \times \frac{5}{20} + \frac{7}{20} \times \frac{7}{20} + \frac{8}{20} + \frac{8}{20} = \frac{138}{400}$ (1)

$P(\text{different colors}) = 1 - P(\text{Same color}) = 1 - \frac{138}{400}$ (1)

$= \frac{262}{400} = \frac{131}{200}$ (1)

7)

Sum 5:	(1,4), (2,3); (3,2); (4,1) (1)	4
Sum 6:	(1,5), (2,4); (3,3); (4,2); (5,1) (1)	5
Sum 7:	(1,6), (2,5); (3,4); (4,3); (5,2); (6,1) (1)	6
Sum 8:	(2,6); (3,5); (4,4); (5,3); (6,2) (1)	5

$P = \frac{20}{36} = \frac{5}{9}$ (1)