This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 03

26JAN2024

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	12M
Teacher :	Bechara Kordahi
Duration :	1 hour

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(30 points)

Consider the transformation T that associates to every point M of affix z the point M' of affix z' such that $z' = (1 + i\sqrt{3})z + \sqrt{3}(1 - i)$.

1) Determine the nature and the elements of T .

2) Calculate the modulus and argument of the following complex ratio :

$\frac{z' - a}{z - a}$ where a is the affix of the center of T .

Question II-(20 points)

Determine the following :

1) $\lim_{x \to 0}\left(\frac{2^x + 4^x - 2}{2x}\right)$ (Answer in the form $aLn2$)

2) $\lim_{x \to +\infty} \frac{\arctan x - \frac{\pi}{2}}{\frac{1}{x}}$

Question III-(20 points)

Determine the n^{th} derivative of the function $y = Lnx^3$ (No need for proof).
Then deduce its derivative of order 25 in its simplest form .

Question IV-(20 points)

Determine the following :

1) $\int \sin 2x.\cos x.dx$

2) $\int \left(\tan^2(2x) + \frac{\cos(Lnx)}{x}\right).dx$

Question V-(10 points)

Find the derivative of : $z = \frac{-Lnx}{x}$. Then , deduce $\int_1^e \frac{Lnx}{x^2}.dx$.

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I $z' = (1+i\sqrt{3})z + \sqrt{3}(1-i)$

1) $a = 1+i\sqrt{3} = 2e^{i\pi/3}$ $\quad z_I = \frac{b}{1-a} = \frac{\sqrt{3}(1-i)}{-i\sqrt{3}} = 1+i$

20 $T = S(I, 2, \frac{\pi}{3})$ where $I(1,1)$

$M \to M'$
$I \to I$

2) $\left|\frac{z'-a}{z-a}\right| = \left|\frac{z_{M'} - z_I}{z_M - z_I}\right| = \frac{IM'}{IM} = K = 2$ — 5

5 $\arg\left(\frac{z'-a}{z-a}\right) = (\overrightarrow{IM}, \overrightarrow{IM'}) = \alpha = \frac{\pi}{3} \ (2\pi)$

II 1) $\lim_{x\to 0} \frac{2^x + 4^x - 2}{2x} = \frac{0}{0} = \lim_{x\to 0} \frac{2^x \ln 2 + 4^x \ln 4}{2} = \frac{3\ln 2}{2}$ — 10

2) $\lim_{x\to +\infty} \frac{\arctan x - \frac{\pi}{2}}{\frac{1}{x}} = \frac{0}{0} = \lim_{x\to +\infty} \frac{\frac{1}{1+x^2}}{\frac{-1}{x^2}} = \lim_{x\to +\infty} \frac{-x^2}{x^2+1}$

10 $= \lim_{x\to +\infty} \frac{-x^2}{x^2} = -1$

III $y = \ln x^3 = 3\ln x$

$y' = \frac{3}{x} = 3x^{-1} = 3(x)^{-1}$

$y'' = -3x^{-2} = 3\left[-(x)^{-2}\right]$

$y''' = 6x^{-3} = 3\left[2(x)^{-3}\right]$ — 15

$y^{IV} = -18x^{-4} = 3\left[-6(x)^{-4}\right]$

$\vdots$

$y^{(n)} = (-1)^{n+1} 3(n-1)! x^{-n}$

Then $y^{(25)} = (-1)^{26}(3)(24!)x^{-25}$

$y^{(25)} = 3(24!)x^{-25} = \frac{3(24!)}{x^{25}}$ — 5

IV 1) $\int \sin 2x \cos x \, dx = \frac{1}{2}\int (\sin 3x + \sin x)\, dx$

$= \frac{1}{2}\left[\frac{-\cos 3x}{3} - \cos x\right] + C = \frac{-1}{2}\left(\frac{\cos 3x}{3} + \cos x\right) + C$

10

OR $\int 2 \sin x \cos^2 x \, dx = -2\int u^2 \, du$ $\quad u = \cos x$

$du = -\sin x \, dx$

$= \frac{-2}{3}\cos^3 x + C$

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2) $\int \left[\tan^2 2x + \frac{\cos(\ln x)}{x}\right] dx = \int \left[1 + \tan^2 2x - 1 + \frac{\cos(\ln x)}{x}\right] dx$

10

$= \frac{\tan(2x)}{2} - x + \sin(\ln x) + C$

V $z = \frac{-\ln x}{x}$ $\quad z' = -\frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{\ln x - 1}{x^2}$ 3

$\int_1^e \frac{\ln x}{x^2} dx = \int_1^e z' dx + \int_1^e \frac{1}{x^2} dx = \left[\frac{-\ln x}{x} - \frac{1}{x}\right]_1^e$

$= \left(\frac{-2}{e}\right) - \left(-1\right) = 1 - \frac{2}{e}$ 7