- This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School
for Boys and Girls Loueizeh - Baabda

SE02 Exam

15DEC2025

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	11S1 and 11S2	
Teacher :	Taline Hawatian	
Duration :	1 hour	
You are to answer the questions to this exam on : Squared paper		
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I.- (9 points)

Consider the family of the curves (C_m): $x^2 + y^2 - 2mx + (m+2)y - 3m - 4 = 0$, where m is a real parameter.

1) Show that, for all m, (C_m) is a circle whose center I_m and radius R are to be determined.
2) Deduce the set of centers I_m of (C_m) as m varies.
3) Show that all the circles pass through two fixed points A and B whose coordinates are to be determined.
4) For $m = 2$, consider the circle (C_2): $x^2 + y^2 - 4x + 4y - 10 = 0$ and the point $F(-5; 0)$. Determine the center and the radius of (C_2).
   Write the equations of the tangents issued from F to (C_2).
5) Among the circles (C_m), are there circles tangent to y-axis?

III.- (8 points)

Determine the following limits.

1) $\lim\limits_{x\to 2} \frac{x-2}{\sqrt{x^2+5}-3}$ 2) $\lim\limits_{x\to 3^+} \frac{\sqrt{x-3}}{x^2-9}$ 3) $\lim\limits_{x\to -\infty} \frac{|x^2+5x|}{\sqrt{x^2+3}}$

III.- (3 points)

Write the equation of the circle passing through the three points $A(0; 4)$, $B(2; -3)$, and $C(5; 3)$.

SE02/20252026/Maths/115152

I. $(C_m): x^2+y^2-2mx+(m+2)y-3m-4=0$

1) $a=\frac{-2m}{-2}=m \quad b=\frac{m+2}{-2} \quad c=-3m-4$

$a^2+b^2-c=m^2+\frac{m^2+4m+4}{4}+3m+4$ 1

$=\frac{4m^2+m^2+4m+4+12m+16}{4}=\frac{5m^2+16m+20}{4}$ 0.5

$5m^2+16m+20=0 \quad \Delta'=64-5(20)=-36<0$ no real roots

$5m^2+16m+20>0$ for all m in $\mathbb{R}$. $a^2+b^2-c>0$ for $m\in\mathbb{R}$.

(C_m) is a circle for all values of m, of center $I_m\left(m,-\frac{m+2}{2}\right)$ and radius $R=\frac{\sqrt{5m^2+16m+20}}{2}$. 0.5

2) $x_I=m \quad y_I=-\frac{m+2}{2} \quad y=-\frac{x+2}{2} \quad x+2y+2=0$. 1

The set of centers I_m is a straight line of equation $x+2y+2=0$

3) $x^2+y^2-2mx+my+2y-3m-4=0$

$x^2+y^2+2y-4+(-2x+y-3)m=0$.

$\begin{cases} x^2+y^2+2y-4=0 \\ -2x+y-3=0 \end{cases} \quad y=2x+3$. 0.5

$x^2+(2x+3)^2+2(2x+3)-4=0$ 0.5

$x^2+4x^2+12x+9+4x+6-4=0 \quad 5x^2+16x+11=0$.

$x_1=-1 \quad y_1=1 \quad A(-1,1)$

$x_2=-\frac{11}{5} \quad y_2=-\frac{22}{5}+3=-\frac{7}{5} \quad B\left(-\frac{11}{5},-\frac{7}{5}\right)$ 1

4) $(C_2): x^2+y^2-4x+4y-10=0 \quad F(-5,0)$

$a=\frac{-4}{-2}=2 \quad b=\frac{4}{-2}=-2 \quad I(2,-2)$

$R=\sqrt{4+4+10}=\sqrt{18}=3\sqrt{2}$

$F(-5,0)$ $\quad 25+20-10=35>0$

$F(-5,0)$ is exterior to the circle.

$(T): y-y_F=m(x-x_F)$ ✓1

$y=m(x+5)$ $\quad mx-y+5m=0$

$d[I/(T)]=R$

$$\frac{|2m+2+5m|}{\sqrt{m^2+1}}=\sqrt{18}$$ ✓1

$|7m+2|=\sqrt{18}\cdot\sqrt{m^2+1}$ square both sides

$(7m+2)^2=18(m^2+1)$ $\quad 49m^2+28m+4-18m^2-18=0$

$31m^2+28m-14=0$ $\quad m_1=\frac{-14+3\sqrt{70}}{31}$ $\quad m_2=\frac{-14-3\sqrt{70}}{31}$

$(T_1): \frac{-14+3\sqrt{70}}{31}x-y+5\left(\frac{-14+3\sqrt{70}}{31}\right)=0$ ✓1

$(T_2): \frac{-14-3\sqrt{70}}{31}x-y+5\left(\frac{-14-3\sqrt{70}}{31}\right)=0$

5) $I_m\left(m,-\frac{m+2}{2}\right)$ $\quad (d): x=0$

$d[I_m/(d)]=R$ $\quad |m|=\frac{\sqrt{5m^2+16m+20}}{2}$ square both sides.

$m^2=\frac{5m^2+16m+20}{4}$ $\quad 4m^2=5m^2+16m+20$

$m^2+16m+20=0$ ✓1

$m_1=-8+2\sqrt{11}$ $\quad m_2=-8-2\sqrt{11}$

Among the circle (C_m), there are 2 circles tangent to y-axis. for $m=-8+2\sqrt{11}$ and $m=-8-2\sqrt{11}$

II- 1) $\lim\limits_{x\to 2}\frac{x-2}{\sqrt{x^2+5}-3}=\frac{0}{0}$ Indeterminate form

$\lim\limits_{x\to 2}\frac{x-2}{\sqrt{x^2+5}-3}\cdot\frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$ ✓2.5

$=\lim\limits_{x\to 2}\frac{(x-2)(\sqrt{x^2+5}+3)}{x^2-4}=\lim\limits_{x\to 2}\frac{(x-2)(\sqrt{x^2+5}+3)}{(x-2)(x+2)}=\frac{6}{4}=\frac{3}{2}$.

2) $\lim_{x\to 3^+} \frac{\sqrt{x-3}}{x^2-9} = \frac{0}{0}$ Indeterminate form

$\lim_{x\to 3^+} \frac{\sqrt{x-3}}{x^2-9} \cdot \frac{\sqrt{x-3}}{\sqrt{x-3}} = \lim_{x\to 3^+} \frac{(x-3)}{(x-3)(x+3)\sqrt{x-3}}$

$= \frac{1}{6\sqrt{0^+}} = \frac{1}{0^+} = +\infty$ 2.5

3) $\lim_{x\to -\infty} \frac{|x^2+5x|}{\sqrt{x^2+3}} = \lim_{x\to -\infty} \frac{|x(x+5)|}{\sqrt{x^2(1+\frac{3}{x^2})}}$

$= \lim_{x\to -\infty} \frac{|x|\cdot|x+5|}{|x|\cdot\sqrt{1+\frac{3}{x^2}}\to 0}$ 3

$= +\infty$

III) $(C): x^2+y^2-2ax-2by+c=0$.

$A(0,4) \in (C)$ $16-8b+c=0$

$B(2,-3) \in (C)$ $4+9-4a+6b+c=0$

$C(5,3) \in (C)$ $25+9-10a-6b+c=0$

$\begin{cases} 8b-c=16 \\ -4a+6b+c=-13 \\ 10a+6b-c=34 \end{cases}$ 1.5

$a=\frac{43}{22}$ $b=\frac{17}{22}$ $c=\frac{-108}{11}$ 1

$(C): x^2+y^2-\frac{43}{11}x-\frac{17}{11}y-\frac{108}{11}=0$ 0.5