This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 02

26MAR2025

Do NOT write in this box. Your MARK will be placed here.

Subject :	A Level : Maths M2Pure
Class :	12A
Teacher :	Bechara Kordahi
Duration :	1 hour

You are to answer the questions to this exam on : Squared paper

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(7 points)

The curve C has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point $P(3, 2)$ lies on C.

Find the equation of the normal to C at the point P, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(7)

Question II-(9 points)

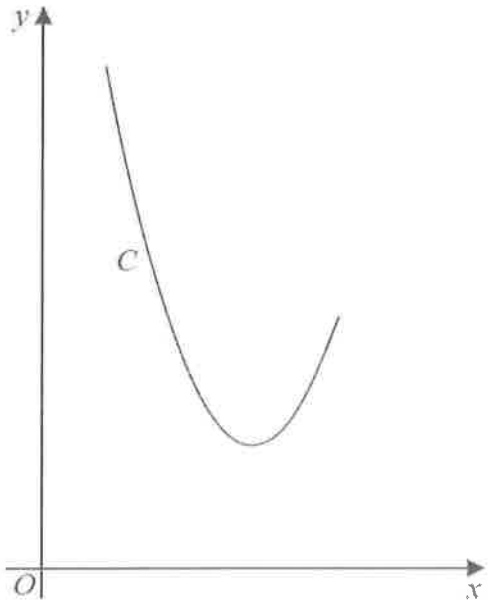


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = f(x)$, where f is a quadratic function.

(b) Find $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants to be found.

(3)

(c) Find the range of f.

(2)

Question III-(8 points)

$$g(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants A, B, C and D.

(5)

A curve has equation

$$y = g(x) \qquad x > 0$$

Using the answer to part (a),

(b) find $g'(x)$.

(2)

(c) Hence, explain why $g'(x) > 3$ for all values of x in the domain of g.

(1)

Question IV-(7 points)

The curve C is defined by the parametric equations

$$x = \frac{1}{t} + 2 \qquad y = \frac{1-2t}{3+t} \qquad t > 0$$

(a) Show that the equation of C can be written in the form $y = g(x)$ where g is the function

$$g(x) = \frac{ax+b}{cx+d} \qquad x > k$$

where a, b, c, d and k are integers to be found.

(5)

(b) Hence, or otherwise, state the range of g.

(2)

Question V-(9 points)

A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The curve crosses the y-axis at the origin and at the point P.

(b) Find the equation of the normal to the curve at P, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

12A

M2 Pure SEQ2

(I) $2x - 4y^2 + 3(x^2 y) = 4x^2 + 8 \qquad P(3,2)$

Then: $2 - 8y\frac{dy}{dx} + 3\left[2xy + \frac{dy}{dx}\cdot x^2\right] = 8x$

Let $m = \left(\frac{dy}{dx}\right)_P \rightarrow 2 - 16m + 3(12 + 9m) = 24$

$2 - 16m + 36 + 27m = 24$, $11m = -14$, $m = -\frac{14}{11}$

gradient normal $= \frac{11}{14}$ $\qquad y - 2 = \frac{11}{14}(x - 3)$ 7

$14y - 28 = 11x - 33 \rightarrow \boxed{-11x + 14y + 5 = 0}$

(II) $\begin{cases} x = 5 + 2\tan t \\ y = 8\sec^2 t \end{cases} \qquad -\frac{\pi}{3} \le t \le \frac{\pi}{4}$

a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{16\sec t \cdot \sec t \tan t}{2\sec^2 t} = 8\tan t$ 4

$x = 3 \rightarrow \tan t = -1 \rightarrow \boxed{\text{gradient}_P = -8}$

b) $y = 8\sec^2 t = 8(1 + \tan^2 t) = 8 + 8\left(\frac{x-5}{2}\right)^2$

Then $\boxed{y = 8 + 2(x-5)^2}$ 3

c) $-\frac{\pi}{3} \le t \le \frac{\pi}{4} \rightarrow -\sqrt{3} \le \tan t \le 1 \rightarrow 0 \le \tan^2 t \le 3$

Then $\boxed{8 \le y \le 32}$ 2

III

a) $g(x) = \frac{3x^3+8x^2-3x-6}{x(x+3)} \equiv Ax+B+\frac{C}{x}+\frac{D}{x+3}$

$Ax^2(x+3)+Bx(x+3)+C(x+3)+Dx \equiv 3x^3+8x^2-3x-6$

$x=0$ $3C=-6 \rightarrow \boxed{C=-2}$, $x=-3$ $-3D=-6$, $\boxed{D=2}$

$x=1$ $4A+4B-8+2=2 \rightarrow A+B=2$ $\boxed{A=3}$ 5

$x=-1$ $2A-2B-4-2=2 \rightarrow A-B=4$ $\boxed{B=-1}$

$2A=6$

b) $y=g(x)$ $x>0$ $g(x)=3x-1-\frac{2}{x}+\frac{2}{x+3}$

$g'(x)=3+\frac{2}{x^2}-\frac{2}{(x+3)^2}=3+\frac{2x^2+12x+18-2x^2}{x^2(x+3)^2}$

$\boxed{g'(x)=3+\frac{6(2x+3)}{x^2(x+3)^2}}$ 2

c) since $x>0$ $\frac{6(2x+3)}{x^2(x+3)^2}>0 \rightarrow \boxed{g'(x)>3}$ 1

IV

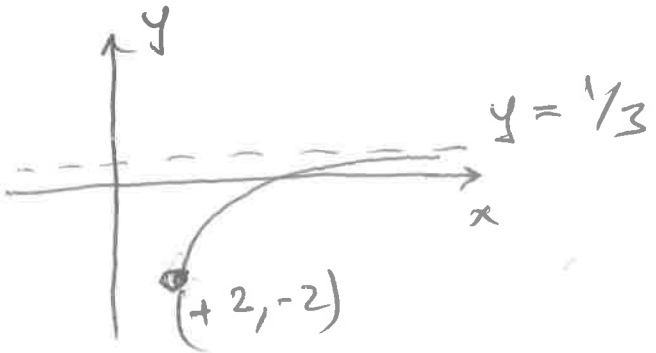
$\begin{cases} x=\frac{1}{t}+2 \\ y=\frac{1-2t}{3+t} \end{cases}$ $t>0$ $\frac{1}{t}=x-2 \rightarrow t=\frac{1}{x-2}$

a) $y=\frac{1-2\left(\frac{1}{x-2}\right)}{3+\frac{1}{x-2}}=\frac{x-2-2}{3x-6+1} \rightarrow \boxed{y=\frac{x-4}{3x-5}}$

$a=1$
$b=-4$
$c=3$
$d=-5$

Also $t>0 \rightarrow x>2 \rightarrow \boxed{k=2}$ 5

b) $\boxed{-2<y<1/3}$ 2



V $4y^2 + 3x = 6(ye^{-2x})$

a) $8y\frac{dy}{dx} + 3 = 6\left[\frac{dy}{dx}e^{-2x} - 2e^{-2x}\cdot y\right]$

$= 6e^{-2x}\frac{dy}{dx} - 12ye^{-2x}$

Then $(6e^{-2x} - 8y)\frac{dy}{dx} = 3 + 12ye^{-2x}$

$$\boxed{\frac{dy}{dx} = \frac{3 + 12ye^{-2x}}{6e^{-2x} - 8y}}$$ 5

b) $x = 0 \rightarrow 4y^2 = 6y \rightarrow 2y(2y-3) = 0 \begin{cases} y_0 = 0 \\ y_P = \frac{3}{2} \end{cases}$

$P(0, \frac{3}{2})$

$\text{gradient}_P = \frac{3+18}{6-12} = \frac{21}{-6} = -\frac{7}{2}$

$\text{gradient}_{\text{normal}} = \frac{2}{7}$ Then $\boxed{y = \frac{2x}{7} + \frac{3}{2}}$ 4