This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 04

27MAR2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	A Level : Maths M2Mechanics
Class :	12A
Teacher :	Bechara Kordahi
Duration :	1 hour

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I

At time t seconds $(t \geq 0)$ a particle has position vector $\mathbf{r}$ meters with respect to a fixed origin O, where $\mathbf{r} = (16t - 3t^3)\mathbf{i} + (t^3 - t^2 + 2)\mathbf{j}$.

(a) Find the velocity of P at the instant when it is moving parallel to the vector $\mathbf{j}$ **(5)**

(b) Find the magnitude of the acceleration of P when $t = 4$ **(4)**

Question II

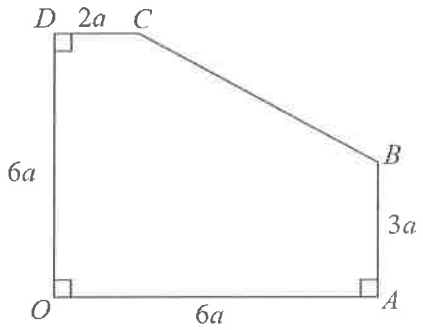


Figure 1

The uniform lamina $OABCD$ is shown in Figure 1, with $OA = 6a$, $AB = 3a$, $CD = 2a$ and $DO = 6a$ and with right angles at O, A and D.

(a) Find the distance of the centre of mass of the lamina

(i) from OD,

(ii) from OA. **(6)**

The lamina is suspended from C and hangs freely in equilibrium with CB inclined at an angle α to the vertical.

(b) Find, to the nearest degree, the size of the angle α. **(4)**

Question III

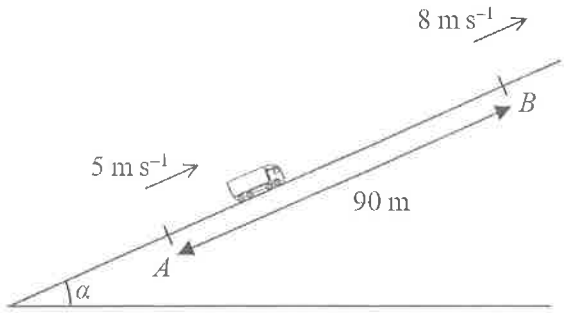


Figure 2

A truck of mass 1200 kg is being driven up a straight road that is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{15}$. The resistance to the motion of the truck from non-gravitational forces is modelled as a single constant force of magnitude 250 N. Two points, A and B, lie on the road, with $AB = 90$ m. The speed of the truck at A is 5 m s^{-1} and the speed of the truck at B is 8 m s^{-1}, as shown in Figure 2.
The truck is modelled as a particle and the road is modelled as a straight line.

(a) Find the work done by the engine of the truck as the truck moves from A to B. **(5)**

On another occasion, the truck is being driven down the same road. The resistance to the motion of the truck is modelled as a single constant force of magnitude 250 N. The engine of the truck is working at a constant rate of 8 kW.
(b) Find the acceleration of the truck at the instant when its speed is 6 m s^{-1}. **(4)**

Question IV

At time $t = 0$ a ball is projected from a fixed point A on horizontal ground to hit a target . The ball is projected from A with speed u at an angle θ to the horizontal .

At time $t = 2$ s the ball hits the target. At the instant when it hits the target the ball is traveling downwards at 30° below the horizontal with speed 12 m s^{-1}.

The ball is modelled as a particle moving freely under gravity and the target is modelled as the point T.

(a) Find the value of θ **and** the value of u (both to 1 decimal place) **(6)**

The height of T above the ground is h meters.

(b) Find the value of h. **(3)**

(c) Find the length of time for which the ball is more than h meters above the ground during the flight from A to T. **(3)**

I $r = (16t - 3t^3)i + (t^3 - t^2 + 2)j \quad t \geq 0$

a) $v = \dot{r} = (16 - 9t^2)i + (3t^2 - 2t)j$ Moving parallel to j

When $16 = 9t^2$, $t^2 = \frac{16}{9} \rightarrow t = \frac{4}{3}$ sec only

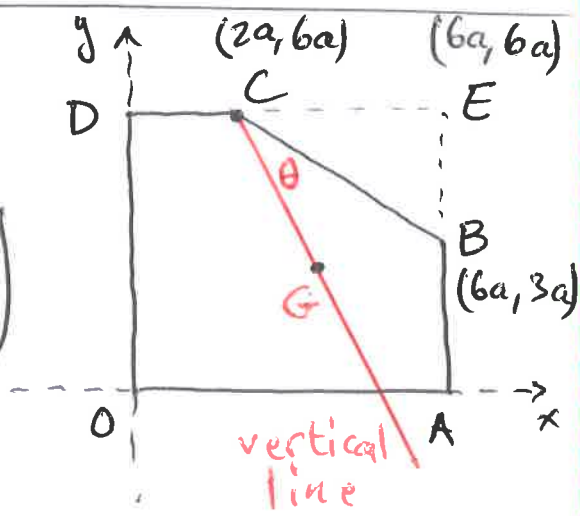
$v = \left[3\left(\frac{4}{3}\right)^2 - 2\left(\frac{4}{3}\right)\right]j = \left(\frac{8}{3}j\right)\text{ms}^{-1}$ (5)

b) $a = \dot{v} = -18ti + (6t - 2)j \quad t = 4\text{sec}, \; a_4 = -72i + 22j$

$|a_4| = \sqrt{(-72)^2 + (22)^2} = \sqrt{5668} \sim 75.3\,\text{ms}^{-2}$ (4)

II a) $\bar{x} = \frac{x_1A_1 + x_2A_2}{A_1 + A_2}$ 1: OAED, 2: BCE

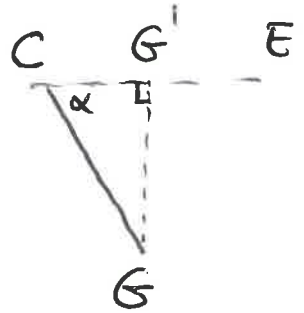
$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{\begin{pmatrix}3a\\3a\end{pmatrix}36a^2 + \begin{pmatrix}14a/3\\5a\end{pmatrix}(-6a^2)}{36a^2 - 6a^2} = \begin{pmatrix}8a/3\\13a/5\end{pmatrix}$ (3)



distance from OD is $8a/3$ (3)

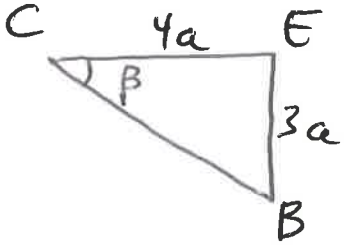
distance from OA is $13a/5$

b) $\tan\alpha = \frac{GG'}{CG'} = \frac{6a - 13a/5}{8a/3 - 2a} = 5.1$



$\alpha = 78.9°$

$\tan\beta = \frac{3a}{4a} = \frac{3}{4}$, $\beta = 36.9°$ (4)



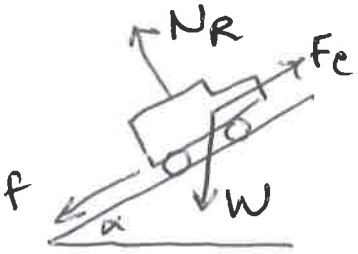
$\theta = \alpha - \beta$, $\theta = 42°$

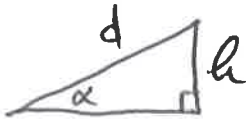
(III) a) $\Delta KE + \Delta PE = Work_f + Work_{Fe}$

$\frac{1}{2}M(v_B^2 - v_A^2) + Mgh = -f \cdot d + Work_{Fe}$

$Work_{Fe} = \frac{1}{2}(1200)(8^2 - 5^2) + 1200(9.8)(6) + 250(90)$

$Work_{Fe} = 116\,460\ J$ (5)

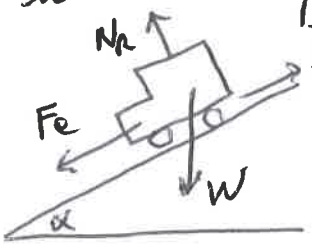




$h = d\sin\alpha = \frac{90}{15} = 6m$

b) $F_e + W\sin\alpha - f = M \cdot a$

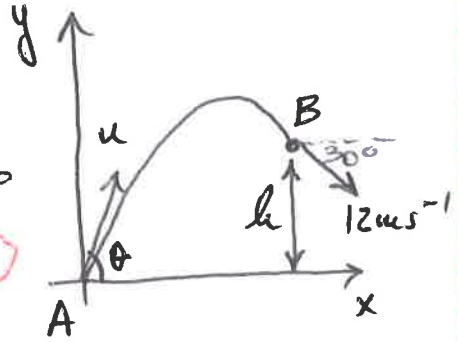
$a = \frac{\frac{8000}{6} + 1200(9.8)/15 - 250}{1200}$, $a = 1.56\ ms^{-2}$ (4)



(IV) a) $u\cos\theta = 12\cos 30^\circ = 12\frac{\sqrt{3}}{2} = 6\sqrt{3}$

$-12\sin 30^\circ = -9.8(2) + u\sin\theta$, $u\sin\theta = 13.6$

$\frac{u\sin\theta}{u\cos\theta} = \frac{13.6}{6\sqrt{3}}$, $\tan\theta = \frac{13.6}{6\sqrt{3}}$, $\theta = 52.6^\circ$ (3)



$u^2\sin^2\theta + u^2\cos^2\theta = 13.6^2 + (6\sqrt{3})^2$

$u^2 = 292.96 \rightarrow u = 17.1\ ms^{-1}$ (3)

b) $y_B = -\frac{1}{2}gt_B^2 + (u\sin\theta)t_B \Rightarrow h = -4.9(2)^2 + 13.6(2)$

then $h = 7.6m$ (3)

c) $y = -\frac{1}{2}gt^2 + (u\sin\theta)t$, $7.6 = -4.9t^2 + 13.6t$

$4.9t^2 - 13.6t + 7.6 = 0$ or $49t^2 - 136t + 76 = 0$

$\Delta = (-136)^2 - 4(49)(76) = 3600$ $\quad t = \frac{136 \pm 60}{98}$

$t_1 = 0.8\ sec$ $\quad t_2 = 2$ 2

$\Delta t = 2 - 0.8 = 1.2\ sec$ 1