This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School
for Boys and Girls — Loucizeh - Baabda

Stage Exam 02
08DEC2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	11SE
Teacher :	Mr. George El Haber
Duration :	1 hour

Calculators are ALLOWED

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I. Given: (E) = $3mx^2 + 4(m + 1)x + 2m = 0$

1) Determine the set of values of m so that the equation (E) admits two real roots x' and x". ($m \neq 0$). [5 pts.]

2) a) Find m so that: $x'^2 + x''^2 = 4$. [5 pts.]

b) Prove that $\frac{1}{x'} + \frac{1}{x''} = \frac{-2m-2}{m}$ [5 pts.]

3) Prove that $m = \frac{-4}{3S+4}$ and P = constant, using S and P. [5 pts.]

II. Find the following limits. [5 pts. each]

1) $\lim_{x \to +\infty} \frac{x^2+2x-3}{3x^2-2x-1}$

2) $\lim_{x \to -1} x^3 - x^2 + x - 1$

3) $\lim_{x \to -\infty} x^2 + x$

4) $\lim_{x \to -\infty} \frac{x^2-3x+2}{x-2}$

5) $\lim_{x \to 0} x - \frac{2}{x+1}$

6) $\lim_{x \to -\infty} \frac{x^2}{x^2+3x}$

7) $\lim_{x \to -\infty} \frac{9x^2-4x}{x+6}$

8) $\lim_{x \to -\infty} \frac{x+2}{x^2-5}$

SE02 Answer key

$3mx^2+4(m+1)x+2m=0$ $\quad a=3m \quad b=4(m+1) \quad c=2m$

1) $\Delta = b^2-4ac$
$=(4m+4)^2-4(3m)(2m)$
$=(4m)^2+2(4m)(4)+4^2-(24m^2)$
$=16m^2+32m+16-24m^2$
$=-8m^2+32m+16$

$\Delta m = b^2-4ac$
$=32^2-4(-8)(16)$
$=1536$

$x' = \dfrac{-b+\sqrt{\Delta}}{2a}$ $\qquad x'' = \dfrac{-b-\sqrt{\Delta}}{2a}$

$= \dfrac{-32+\sqrt{1536}}{2(-8)}$ $\qquad = \dfrac{-32-\sqrt{1536}}{2(-8)}$

$= 2-\sqrt{6} \simeq -0.44$ $\qquad = 2+\sqrt{6} \simeq 4.5$

m	$-\infty$		$2-\sqrt{6}$		$2+\sqrt{6}$		$+\infty$
Δ		$-$	0	$+$	0	$-$	

$m \in]2-\sqrt{6}, 2+\sqrt{6}[$

2) a) $x'^2 + x''^2 = 4$

$P = \frac{c}{a} = \frac{2m}{3m}$

$S = -\frac{b}{a} = \frac{-4(m+1)}{3m}$

$(x' + x'')^2 = x'^2 + 2(x')(x'') + x''^2$

$S^2 = x'^2 + x''^2 + 2P$

$S^2 = 4 + 2P$

$S^2 - 2P = 4$

$\left(\frac{-4m-4}{3m}\right)^2 - 2\left(\frac{2m}{3m}\right) = 4$

$\left(\frac{-4m-4}{3m}\right)^2 - 2\left(\frac{2}{3}\right) = 4$

$\frac{(4m+4)^2}{9m^2} - \frac{4}{3} = 4$

$\frac{16m^2+32m+16}{9m^2} - \frac{4}{3} - 4 = 0$

$\frac{16m^2+32m+16}{9m^2} - \frac{16}{3} = 0$

$\frac{16m^2+32m+16-48m^2}{9m^2} = 0$

$\frac{-32m^2+32m+16}{9m^2} = 0$

$m' = \frac{1-\sqrt{3}}{2}$; $m'' = \frac{1+\sqrt{3}}{2}$

b) $\frac{1}{x'} + \frac{1}{x''} = \frac{-2m-2}{m}$

$\frac{x''+x'}{x''x'} = \frac{S}{P}$

$= \frac{\frac{-4(m+1)}{3m}}{\frac{2m}{3m}} = \frac{-4(m+1)}{2m} = \frac{-4m-4}{2m} = \frac{-2m-2}{m}$

Verified!

3) $S = \frac{-4(m+1)}{3m}$

$3mS = -4m-4$

$3mS+4m = -4$

$(3S+4)m = -4$

$m = \frac{-4}{3S+4}$ Verified

$P = \frac{2m}{3m} = \frac{2}{3}$ Verified

II- 1) $\lim_{x \to +\infty} \frac{x^2+2x-3}{3x^2-2x-1} = \frac{1}{3}$

2) $\lim_{x \to -1} x^3 - x^2 + x - 1 = -4$

3) $\lim_{x \to -\infty} x^2 + x = +\infty$

4) $\lim_{x \to -\infty} \frac{x^2-3x+2}{x-2} = -\infty$

5) $\lim_{x \to 0} x - \frac{2}{x+1} = -2$

6) $\lim_{x \to -\infty} \frac{x^2}{x^2+3x} = 1$

7) $\lim_{x \to -\infty} \frac{9x^2-4x}{x+6} = -\infty$

8) $\lim_{x \to -\infty} \frac{x+2}{x^2-5} = 0$