This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 03

16MAY2025

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths	
Class :	11S	
Teacher :	Taline Hawatian	
Duration :	1 hour	
You are to answer the questions to this exam on : Squared paper		
Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I.-(5 points)

Given that $cosx = -\frac{\sqrt{5}}{4}$ such that $x \in]0;\pi[$. Without calculating x, calculate the exact value of $cos2x, sin2x, tan2x$.

II.- (8 points)

1) Solve the following equations:

   a) $cos3x = sin\left(\frac{\pi}{4}+x\right)$ $x \in \left]-\frac{\pi}{2};\frac{3\pi}{2}\right[$.

   b) $cosx + \sqrt{3}sinx = \sqrt{3}$.

2) Prove that $\frac{sin5x-2sin3x+sinx}{cos5x-cosx} = tanx$.

III.- (6 points)

1) Express $cos\left(\frac{\pi}{4}+x\right)$ and $sin\left(\frac{\pi}{4}-x\right)$ in terms of $sinx$ and $cosx$.
2) Show that $cos\left(\frac{\pi}{4}+x\right)sin\left(\frac{\pi}{4}-x\right) = \frac{1}{2} - \frac{sin2x}{2}$
3) Deduce the value of $cos\frac{5\pi}{12}sin\frac{\pi}{12}$.
4) Solve the equation $2cos\left(\frac{\pi}{4}+x\right)sin\left(\frac{\pi}{4}-x\right) = \frac{1}{2}$

IV.- (6 points)

Given in an orthonormal system $(O;\ \vec{i};\ \vec{j};\ \vec{k})$, the points $A(5;4;-3), B(-1;2;-4)$, $C(2;8;-5)$, and $D(1;2;2)$.

1) Calculate the components of vectors $\overrightarrow{AB}, \overrightarrow{AC}$, and $\overrightarrow{AD}$.
2) Show that the points A, B, C, and D are not coplanar.
3) Calculate the coordinates of the center of gravity of triangle ABC.
4) Given the point $I(\frac{1}{2};5;-\frac{9}{2})$, show that A, I, G are collinear points.
5) Calculate the coordinates of the vector $\vec{u}(-7;4;2)$ in the basis $(\overrightarrow{AB};\ \overrightarrow{AC};\ \overrightarrow{AD})$.

SE03/20242025/Maths/11S

I.- $\cos x = -\frac{\sqrt{5}}{4}$ $x \in]0, \pi[$ $x \in QI$ or QII 0.5

$\cos x < 0$ $x \in QII$

$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{5}{16}\right) - 1 = -\frac{3}{8}$ 1

$\sin^2 x = 1 - \cos^2 x = 1 - \frac{5}{16} = \frac{11}{16}$ 1

$\sin x = \frac{\sqrt{11}}{4}$ accepted 0.5 $\sin x = -\frac{\sqrt{11}}{4}$ rejected

$\sin 2x = 2\sin x \cos x = 2\left(\frac{\sqrt{11}}{4}\right)\left(-\frac{\sqrt{5}}{4}\right) = \frac{-\sqrt{55}}{8}$ 1

$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{\frac{-2\sqrt{55}}{16}}{\frac{-6}{16}} = \frac{\sqrt{55}}{3}$ 1

II.- 1) a) $\cos 3x = \sin\left(\frac{\pi}{4} + x\right)$

$\sin\left(\frac{\pi}{2} - 3x\right) = \sin\left(\frac{\pi}{4} + x\right)$ 0.5

$\frac{\pi}{2} - 3x = \frac{\pi}{4} + x + 2k\pi$ 0.5 OR $\frac{\pi}{2} - 3x = \pi - \left(\frac{\pi}{4} + x\right) + 2k\pi$ 0.5

$-3x - x = \frac{\pi}{4} - \frac{\pi}{2} + 2k\pi$ $\frac{\pi}{2} - 3x = \frac{3\pi}{4} - x + 2k\pi$

$-4x = -\frac{\pi}{4} + 2k\pi$ 0.5 $-2x = \frac{\pi}{4} + 2k\pi$

$x = \frac{\pi}{16} - \frac{k\pi}{2}$ $x = -\frac{\pi}{8} + k\pi$

$k = 0$ $x = \frac{\pi}{16}$ $k = 0$ $x = -\frac{\pi}{8}$

$k = -1$ $x = \frac{9\pi}{16}$ $k = 1$ $x = \frac{7\pi}{8}$

$k = -2$ $x = \frac{17\pi}{16}$

$k = 1$ $x = -\frac{7\pi}{16}$

$S = \left\{-\frac{7\pi}{16}, \frac{\pi}{16}, \frac{9\pi}{16}, \frac{17\pi}{16}, -\frac{\pi}{8}, \frac{7\pi}{8}\right\}$ 1

b) $\cos x + \sqrt{3}\sin x = \sqrt{3}$

$\cos x + \tan\frac{\pi}{3}\sin x = \sqrt{3}$ 0.5

$\left(\cos x + \frac{\sin\pi/3}{\cos\pi/3}\sin x = \sqrt{3}\right)\left(\cos\frac{\pi}{3}\right)$

$\cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x = \sqrt{3}\cos\frac{\pi}{3}$ 0.5

$\cos\left(x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$ 0.5

$x - \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi$ 0.5 $\qquad x = \frac{\pi}{2} + 2k\pi$

$x - \frac{\pi}{3} = -\frac{\pi}{6} + 2k\pi$ 0.5 $\qquad x = \frac{\pi}{6} + 2k\pi$

2) $\frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x - \cos x}$

$= \frac{2\sin 3x\cos 2x - 2\sin 3x}{-2\sin 3x\sin 2x}$ 1

$= \frac{2\sin 3x(\cos 2x - 1)}{-2\sin 3x\sin 2x}$ 0.5 $= \frac{-2\sin^2 x}{-2\sin x\cos x}$ 0.5 0.5

$= \frac{\sin x}{\cos x} = \tan x$ verified

III- 1) $\cos\left(\frac{\pi}{4} + x\right) = \cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x$ 0.5 0.25

$= \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$

$\sin\left(\frac{\pi}{4} - x\right) = \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x$ 0.5 0.25

$= \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$

2) $\cos\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right)$ 0.5

$= \frac{\sqrt{2}}{2}(\cos x - \sin x)\left(\frac{\sqrt{2}}{2}\right)(\cos x - \sin x)$ 0.5

$= \frac{1}{2}(\cos x - \sin x)^2 = \frac{1}{2}(\cos^2 x + \sin^2 x - 2\sin x\cos x)$

$= \frac{1}{2} - \sin x\cos x = \frac{1}{2} - \frac{\sin 2x}{2}$ 0.5 verified

3) $\cos\frac{5\pi}{12}\sin\frac{\pi}{12} = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$ 1

$= \frac{1}{2} - \frac{\sin\pi/3}{2} = \frac{1}{2} - \frac{\sqrt{3}}{4} = \frac{2-\sqrt{3}}{4}$

4) $2\cos\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right) = \frac{1}{2}$

$\cos\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right) = \frac{1}{4}$ 0.5

$\frac{1}{2} - \frac{\sin 2x}{2} = \frac{1}{4} \qquad -\frac{\sin 2x}{2} = -\frac{1}{4}$

$\sin 2x = \frac{1}{2} = \sin\frac{\pi}{6}$ 0.5

$2x = \frac{\pi}{6} + 2k\pi$ or $2x = \pi - \frac{\pi}{6} + 2k\pi$

$x = \frac{\pi}{12} + k\pi$ 0.5 $\qquad x = \frac{5\pi}{12} + k\pi$ 0.5

IV. $A(5;4;-3)$ $B(-1;2;-4)$ $C(2;8;-5)$ $D(1;2;2)$

1) $\vec{AB}(-6;-2;-1)$ $\vec{AC}(-3;4;-2)$ $\vec{AD}(-4;-2;5)$ 0.75

2) $\vec{AD} = m\vec{AB} + n\vec{AC}$ 0.5

$\begin{cases} -4 = -6m - 3n \\ -2 = -2m + 4n \\ 5 = -m - 2n \end{cases}$ 0.5 $\begin{cases} -2m+4n=-2 \\ -m-2n=5 \end{cases}$

$m = -2 \qquad n = -\frac{3}{2}$ 0.25

$-6(-2) - 3\left(-\frac{3}{2}\right) \stackrel{?}{=} -4$ 0.5 $\frac{33}{2} \neq -4$

$\Rightarrow \vec{AB}, \vec{AC}$, and $\vec{AD}$ are not coplanar vectors

Therefore A, B, C, D are not coplanar points.

3) $x_G = \frac{x_A+x_B+x_C}{3} = \frac{5-1+2}{3} = 2$ 0.5

$y_G = \frac{y_A+y_B+y_C}{3} = \frac{4+2+8}{3} = \frac{14}{3}$ 0.5

$z_G = \frac{z_A+z_B+z_C}{3} = \frac{-3-4-5}{3} = -4$ 0.5

$G\left(2;\frac{14}{3};-4\right)$

4) $I\left(\frac{1}{2}; 5; -\frac{9}{2}\right)$ $G\left(2; \frac{14}{3}; -4\right)$ $A(5; 4; -3)$

$\vec{AI}\left(-\frac{9}{2}; 1; -\frac{3}{2}\right)$ $\vec{AG}\left(-3; \frac{2}{3}; -1\right)$ 1

$\vec{AI} = \frac{3}{2}\vec{AG}$ $\Rightarrow$ A, I, G are collinear points.

5) $\vec{u} = m\vec{AB} + n\vec{AC} + p\vec{AD}$

$$\begin{cases} -7 = -6m - 3n - 4p \\ 4 = -2m + 4n - 2p \\ 2 = -m - 2n + 5p \end{cases}$$ 0.5

$m = -\frac{6}{41}$ 0.5

$n = \frac{57}{41}$ $p = \frac{38}{41}$

$\vec{u}\left(\frac{-6}{41}; \frac{57}{41}; \frac{38}{41}\right)$