This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 03

26JAN2024

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	11S
Teacher :	Taline Hawatian
Duration :	1 hour

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I.- (14 points)

Designate by f the function defined over R-{0} by $f(x) = \frac{x^3 - x^2 - x + 1}{x^2}$ and (C) its representative curve in an orthonormal system (O; $\vec{\imath}$; $\vec{\jmath}$).

1) Calculate the limits of f at the endpoints of its domain of definition.

   Deduce an asymptote to (C).

2) a) Determine the real numbers a, b, c so that: $f(x) = ax + b + \frac{cx+1}{x^2}$.

   b) Show that the straight line (L): $y = x - 1$ is an asymptote to (C).

   c) Calculate the coordinates of the point of intersection of (C) and (L).

   Study the relative position of the straight line (L) with respect to the curve (C).

3) Show that $f'(x) = \frac{x(x^3+x-2)}{x^4}$ and study its sign. Set up the table of variations of f.

4) Determine the points of intersection of (C) with the x-axis.

5) Write the equation of the tangent to (C) at the point A of abscissa 2.

6) Calculate $f(-2), f(0.5), f(3)$.

7) Plot (C) in $(O, \vec{\imath}, \vec{\jmath})$.

8) Solve, graphically, the equation $f(x) > 0$.

9) Discuss, according to the values of m, the number and sign of solutions of the equation $f(x) = m$.

II.- (6 pts)

Designate by f_m the function defined over R by $f_m : x \to mx^2 + (3m + 1)x - 4m$, $m \in R - \{0\}$, of representative curve (C_m).

1) Show that the curves (C_m) pass through two fixed points to be determined.

2) Show that the curve (C_m) cuts the x-axis for any non-zero real number m.

SE03/20232024/Maths/11S

I.- $f(x) = \frac{x^3-x^2-x+1}{x^2}$ $\quad D_f =]-\infty;0[\cup]0;+\infty[$

1) $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \frac{x^3-x^2-x+1}{x^2} = \lim\limits_{x\to+\infty} \frac{x^3}{x^2} = \lim\limits_{x\to+\infty} x = +\infty$ ✓ 0.5

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \frac{x^3}{x^2} = \lim\limits_{x\to-\infty} x = -\infty$

$\lim\limits_{x\to0} f(x) = \lim\limits_{x\to0} \frac{x^3-x^2-x+1}{x^2} = \frac{1}{0+} = +\infty$ 0.5 ✓ 0.5

$x=1$ is a vertical asymptote.

2) a) $f(x) = ax+b+\frac{cx+1}{x^2}$

$= \frac{ax^3+bx^2+cx+1}{x^2} = \frac{x^3-x^2-x+1}{x^2}$

$ax^3 = x^3$ $\quad bx^2 = -x^2$ $\quad cx = -x$ ✓ 1

$a=1$ $\quad b=-1$ $\quad c=-1$

$f(x) = x+1+\frac{1-x}{x^2}$

b) $\lim\limits_{x\to\pm\infty} [f(x)-(x-1)] = \lim\limits_{x\to\pm\infty} [x-1+\frac{1-x}{x^2}-x+1]$ ✓ 1

$= \lim\limits_{x\to\pm\infty} \frac{1-x}{x^2} = \lim\limits_{x\to\pm\infty} \frac{-x}{x^2} = \frac{-1}{\pm\infty} = 0$

$\Rightarrow (L): y = x-1$ is an oblique asymptote to (C) at $\pm\infty$.

c) $y_{(C)} = y_{(L)}$

$x-1+\frac{1-x}{x^2} = x-1$

$\frac{1-x}{x^2} = 0 \quad 1-x=0 \quad x=1 \quad y=0$ ✓ 0.5

$(1;0)$ is the point of intersection of (C) and (L)

c) $f(x) - y_{(L)} = \frac{1-x}{x^2}$

$f(x) - y_{(L)} = 0 \quad \frac{1-x}{x^2} = 0 \quad x = 1$. (C) cuts (L) at $(1;0)$

$f(x) - y_{(L)} < 0 \quad 1 - x < 0 \quad x > 1$ ✓0.5

The curve (C) is below the straight line (L) for $x > 1$.

$f(x) - y_{(L)} > 0 \quad 1 - x > 0 \quad x < 1$ ✓0.5

The curve (C) is above the straight line (L) for $x < 1$.

3) $f'(x) = \frac{(3x^2 - 2x - 1)(x^2) - 2x(x^3 - x^2 - x + 1)}{x^4}$

$= \frac{3x^4 - 2x^3 - x^2 - 2x^4 + 2x^3 + 2x^2 - 2x}{x^4}$ ✓1.5

$= \frac{x^4 + x^2 - 2x}{x^4} = \frac{x(x^3 + x - 2)}{x^4}$

$f'(x) = 0 \quad x(x^3 + x - 2) = 0 \quad x \neq 0 \; ; \; x = 1 \; , y = 0$

$f'(x) > 0$ for $x < 0$ and $x > 1$ ✓0.5

$f'(x) < 0$ for $0 < x < 1$

x	$-\infty$		0		1		$+\infty$
f'		$+$	‖	$-$	0	$+$	
f	$-\infty$	↗	$+\infty$ ‖ $+\infty$	↘	0 (min. pt)	↗	$+\infty$

✓1

4) $y = 0 \quad x^3 - x^2 - x + 1 = 0 \quad x = -1 \quad x = 1$ ✓0.5

$(-1;0) \quad (1;0)$

5) (T): $y - f(2) = f'(2)(x - 2)$ ✓1

$f(2) = \frac{3}{4} \quad f'(2) = 1 \quad y - \frac{3}{4} = x - 2 \quad (T): y = x - \frac{5}{4}$

6) $f(-2) = -\frac{9}{4} \quad f(0.5) = 1.5 \quad f(3) = \frac{16}{9}$ ✓1

7) Figure

8) $f(x)>0$ for $x \in]-1;0[\cup]0;1[\cup]1;+\infty[$ 0.5

9) $f(x)=m$

for $m<0$ 1 negative roots

for $m=0$ 2 roots, 1 positive and 1 negative 1.5

$x=-1$, $x=1$

for $m>0$ 3 roots, 1 negative and 2 positive

II- $f_m : x \longrightarrow mx^2+(3m+1)x-4m$

1) $mx^2+(3m+1)x-4m=y$

$mx^2+3mx+x-4m-y=0$ 1

$(x^2+3x-4)m+x-y=0$

$\begin{cases} x^2+3x-4=0 \\ x-y=0 \end{cases}$ 0.5 $\quad x=1 \quad x=-4$ $\quad y=1 \quad y=-4$ 1.5

$(-4;-4)$ and $(1;1)$ are the fixed points.

2) $mx^2+(3m+1)x-4m=0$

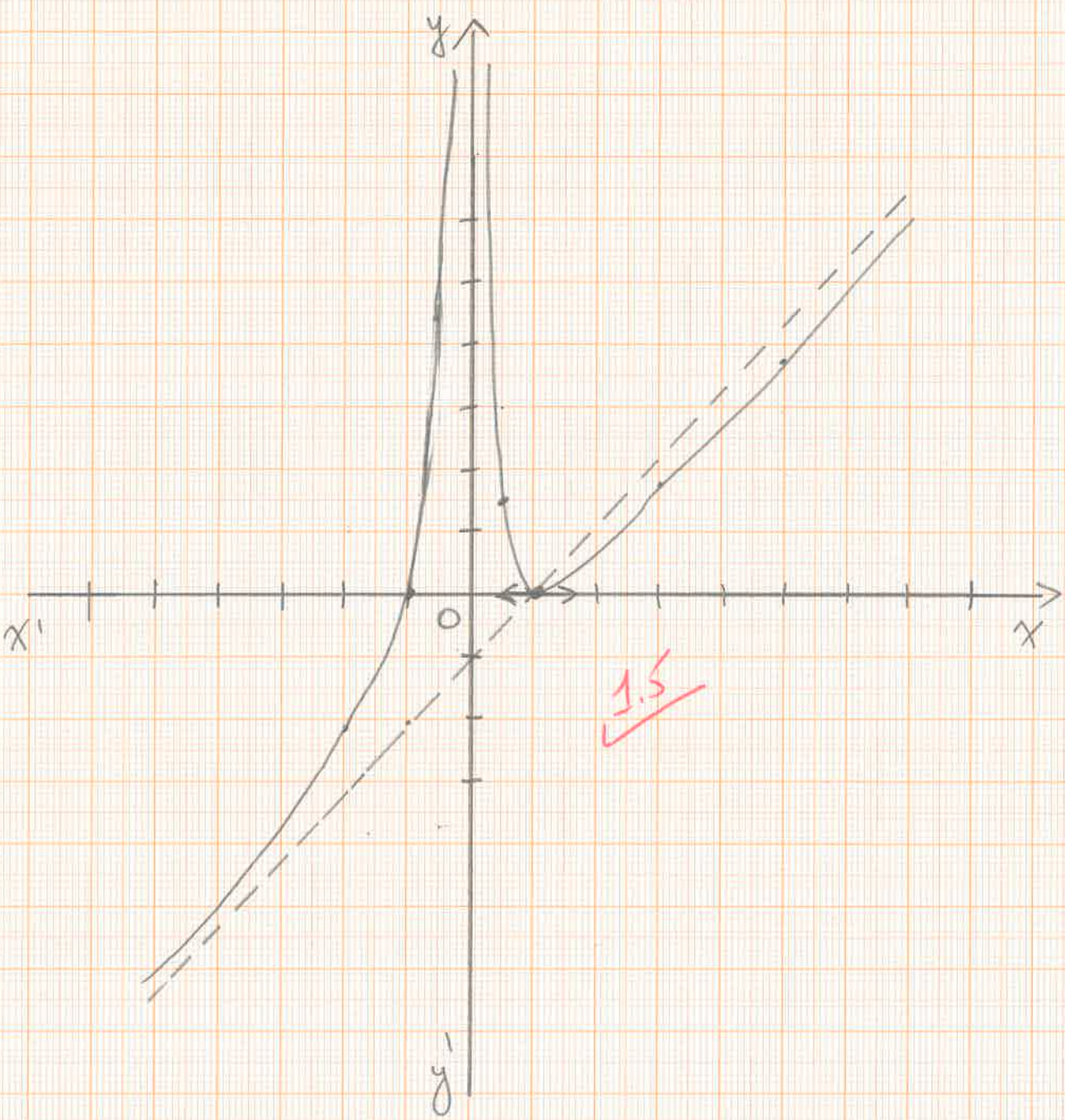
$\Delta = (3m+1)^2-4m(-4m)$ 1

$= (3m+1)^2+16m^2$

$\Delta>0$ for all $m \in \mathbb{R}^*$ 1

The equation admits 2 real roots for all $m \in \mathbb{R}^*$ 1

(C_m) cuts the x-axis in 2 points for any non-zero m.



1.5

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