This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 01

11NOV2022

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	12M
Teacher :	Bechara Kordahi
Duration :	2 hours

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(40 points)

In the table below, only one of the proposed answers to each question, is correct. Write down the number of each question and give, **with justification**, the answer corresponding to it.

N°	Questions	Answers			
		a	b	c	d
1	$z=-2e^{-i\frac{5\pi}{6}}$. An argument of z is :	$\frac{-\pi}{6}$	$\frac{\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{6}$
2	The solution set of the inequality $Ln(x^2-2x+2)>0$ is :	IR	$]0;+\infty[$	$IR-\{1\}$	$]1;+\infty[$
3	$\lim\limits_{x\to+\infty} x.Ln\left(1+\frac{1}{x}\right)=$	0	1	$+\infty$	$-\infty$
4	z and z ' are two complex numbers. If $z'=\frac{\bar{z}-i}{2z+2i}$ then $\lvert z'\rvert=$	$\lvert z\rvert$	2	$\lvert z\rvert\times\lvert\bar{z}\rvert$	$\frac{1}{2}$

Question II-(40 points)

The complex plane is referred to a direct orthonormal system $(O ; \vec{u}, \vec{v})$.

To every point M(x ; y) with affix $z \neq 0$, associate the point M′ (x′ ; y′) with affix z′ so that $z' = z + \frac{1}{z}$.

1) Prove that $x' = x\left(1 + \frac{1}{x^2 + y^2}\right)$ and $y' = y\left(1 - \frac{1}{x^2 + y^2}\right)$.

2) Assume that M moves on the circle with equation $x^2 + y^2 = 4$.
Express x′ and y′ in terms of x and y. then prove that M′ moves on the ellipse (E) with equation $36x^2 + 100y^2 = 225$.

3) Assume that M moves on the line with equation $y = x$, deprived from O.
Verify that M′ moves on the hyperbola (H) with equation $x^2 - y^2 = 2$.

4) Prove that the ellipse (E) and the hyperbola (H) have the same foci.

5) Let $A\left(\frac{5\sqrt{2}}{4}, \frac{3\sqrt{2}}{4}\right)$ be one of the common points of (E) and (H).
Prove that the tangents at A to (E) and (H) are perpendicular.

Question III-(40 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A and B such that : $z_A = 1$ and $z_B = \frac{3}{2} + i\frac{\sqrt{3}}{2}$.
Let (C) be the circle with center A and radius 1.

1) a - Write $z_B - z_A$ in the exponential form.

b – Determine a measure of the angle $(\vec{u}; \vec{AB})$.

c – Show that the point B belongs to the circle (C).

2) To every point M, of non-zero affix z, associate the point M' of affix z' such that : $z' = \frac{\bar{z} + 2}{\bar{z}}$.

a – Prove that $\bar{z}(z' - 1) = 2$.

b – Deduce, when M' moves on the circle (C), that M moves on a circle (T) to be determined.

Question IV-(40 points)

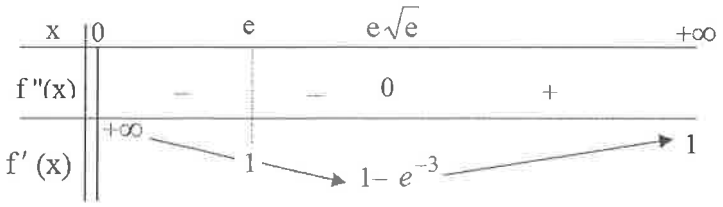
Let f be the function defined, on $]0 ; +\infty[$ by $f(x) = x + 2\dfrac{Lnx}{x}$. (C) is the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$; unit 2 cm .

1) a – Calculate $\lim\limits_{x \to 0} f(x)$ and give its graphical interpretation.

b – Determine $\lim\limits_{x \to +\infty} f(x)$ and verify that the line (d) of equation $y = x$ is an asymptote of (C) .

c – Study according to the values of x , the relative position of (C) and (d) .

2) The table below is the table of variations of the function f ', the derivative of f.



a – Show that f is strictly increasing on its domain of definition, and set up its table of variations.

b – Write an equation of the line (D) that is tangent to (C) at the point G of abscissa e .

c – Prove that the curve (C) has a point of inflection L to be determined .

d – Show that the equation $f(x) = 0$ has a unique root α and verify that :

$$0.75 < \alpha < 0.76 .$$

3) Draw (D) , (d) and (C) .

12M

SE01 Mathematics correction

Nov, 2022

I 1) $z = -2e^{-i\frac{5\pi}{6}} = 2e^{i\pi/6}$ $\boxed{\arg(z) = \frac{\pi}{6}}$ (b) 10

2) $\ln(x^2-2x+2) > 0$ $x^2-2x+2 > 0$ $a>0$ $\Delta' = 1-2 < 0$ for all $x \in \mathbb{R}$

$x^2-2x+2 > 1$, $x^2-2x+1 > 0$, $(x-1)^2 > 0$, $\boxed{x \neq 1}$ (c) 10

3) $\lim_{x\to+\infty} x \ln\left(1+\frac{1}{x}\right) = \lim_{x\to+\infty} \frac{\ln(1+1/x)}{1/x} = \frac{0}{0} =$

$$\lim_{x\to+\infty} \frac{\frac{-1/x^2}{1+1/x}}{\frac{-1/x^2}{1}} = \frac{1}{1+0} = \boxed{1}$$ (b) 10

4) $|z'| = \frac{|\bar{z}-i|}{|2(z+i)|} = \frac{|\overline{z+i}|}{2|z+i|} = \frac{|z+i|}{2|z+i|} = \boxed{\frac{1}{2}}$ (d) 10

II $z' = z + \frac{1}{z}$ $z = x+iy = z_M$, $z' = x'+iy' = z_{M'}$ $z \neq 0$

1) $x'+iy' = x+iy + \frac{1}{x+iy}\cdot\frac{x-iy}{x-iy} = x+iy + \frac{x-iy}{x^2+y^2}$

$$x'+iy' = x + \frac{x}{x^2+y^2} + i\left(y - \frac{y}{x^2+y^2}\right) = x\left(1+\frac{1}{x^2+y^2}\right) + y\left(1-\frac{1}{x^2+y^2}\right)i$$

then $\boxed{x' = x\left(1+\frac{1}{x^2+y^2}\right) \quad y' = y\left(1-\frac{1}{x^2+y^2}\right)}$ 8

2) $x^2+y^2=4$ $\begin{cases} x' = 5x/4 \to x = \frac{4}{5}x' \\ y' = 3y/4 \to y = \frac{4}{3}y' \end{cases}$ $x^2+y^2=4$

$\frac{16}{25}x'^2 + \frac{16}{9}y'^2 = 4$, $144x'^2 + 400y'^2 = 900$ divide by 4

hence $\boxed{36x'^2 + 100y'^2 = 225}$ 8

3) $y = x$ $\begin{cases} x' = x\left(1 + \frac{1}{2x^2}\right) = x + \frac{1}{2x} \\ y' = x\left(1 - \frac{1}{2x^2}\right) = x - \frac{1}{2x} \end{cases}$ 8

$x'^2 - y'^2 = x^2 + 1 + \frac{1}{4x^2} - x^2 + 1 - \frac{1}{4x^2} = 2$ or $x^2 - y^2 = 2$ verified

4) (E) and (H) have the same center O.

$(E): \frac{x^2}{25/4} + \frac{y^2}{9/4} = 1$ $\quad c^2 = \frac{25}{4} - \frac{9}{4} = \frac{16}{4} = 4$, $c = 2$ 8

$(H): \frac{x^2}{2} - \frac{y^2}{2} = 1$ $\quad c^2 = 2 + 2 = 4$, $c = 2$ verified

5) $A\left(\frac{5\sqrt{2}}{4}, \frac{3\sqrt{2}}{4}\right) \in (E \cap H)$ 8

$36x^2 + 100y^2 = 225$

$72x + 200yy' = 0$

$y' = \frac{-72}{200}\frac{x}{y} = \frac{-9}{25}\frac{x}{y}$

$x^2 - y^2 = 2, \quad 2x - 2yy' = 0$

$y' = \frac{x}{y}$

$(\text{slope}_A)_E \cdot (\text{slope}_A)_H$

$= \frac{-9}{25} \cdot \frac{\frac{5\sqrt{2}}{4}}{\frac{3\sqrt{2}}{4}} \cdot \frac{\frac{5\sqrt{2}}{4}}{\frac{3\sqrt{2}}{4}} = \frac{-9(50)}{25(18)} = -1$ verified

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III $z_A = 1$ $\quad z_B = \frac{3}{2} + \frac{i\sqrt{3}}{2}$ $\quad C(A, 1)$

1) (a) $z_B - z_A = \frac{3}{2} + \frac{i\sqrt{3}}{2} - 1 = \frac{1}{2} + \frac{i\sqrt{3}}{2} = \frac{1}{2}(1 + i\sqrt{3})$

$= \frac{1}{2} 2e^{i\pi/3} = e^{i\pi/3}$ 10

(b) $(\vec{u}, \vec{AB}) = \arg(z_{\vec{AB}}) = \arg(z_B - z_A) = \frac{\pi}{3}\ (2\pi)$ 5

(c) $|z_B - z_A| = 1 = AB = r_{(C)}$ verified (*) 5

2) $z' = \frac{\bar{z} + 2}{\bar{z}}$ (a) $\bar{z}(z' - 1) = \bar{z}\left(\frac{\bar{z} + 2}{\bar{z}} - 1\right) = 2$ (*) 5

(b) $M' \in (C) \Rightarrow AM' = 1, \ |z_{M'} - z_A| = 1$ or $|z' - 1| = 1$

$|\bar{z}(z'-1)| = 2$, $|\bar{z}| \cdot |z'-1| = 2$, $|z|(1) = 2$, $|z| = 2$

M ∈ the circle (T) of center O radius 2. (*)

15

IV (C) : $f(x) = x + \frac{2\ln x}{x}$ $D_f =]0, +\infty[$ 1 unit = 2 cm

1) (a) $\lim_{x \to 0^+} f(x) = 0 - \frac{\infty}{0^+} = -\infty$ 3 $x = 0$ VA 2

(b) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left(x + \frac{2\ln x}{x}\right) = +\infty$ 3 (*) 2

$\lim_{x \to +\infty} [f(x) - x] = \lim_{x \to +\infty} \frac{2\ln x}{x} = 0^+$ verified

(c) $g(x) = f(x) - x = \frac{2\ln x}{x}$ 3 (*)

$g(x) > 0$, $x > 1$ (C) above (d)

$g(x) < 0$, $0 < x < 1$ (C) below (d)

$g(x) = 0$, $x = 1$ (C) cuts (d) at (1, 1)

2) (a) $f'_{min} = 1 - e^{-3} \approx 0.95 > 0 \Rightarrow f'(x) > 0$, (*) f: strictly increases 3

x	0		$+\infty$
f'		+	
f	$-\infty$	↗	$+\infty$

(*) 4

(b) $x_G = e$ $y_G = e + \frac{2}{e}$ slope $= f'(e) = 1$

(D) 5

$y - \left(e + \frac{2}{e}\right) = 1(x - e) \Rightarrow$ (D) : $y = x + \frac{2}{e}$

(c) $f''(x) = 0 \Rightarrow x_L = e\sqrt{e} = e^{3/2}$, $y_L = e^{3/2} + \frac{3}{e^{3/2}}$ 3 (*)

(C) has a point of inflection L around which $f''(x)$ changes its sign. (*) 2

3/4

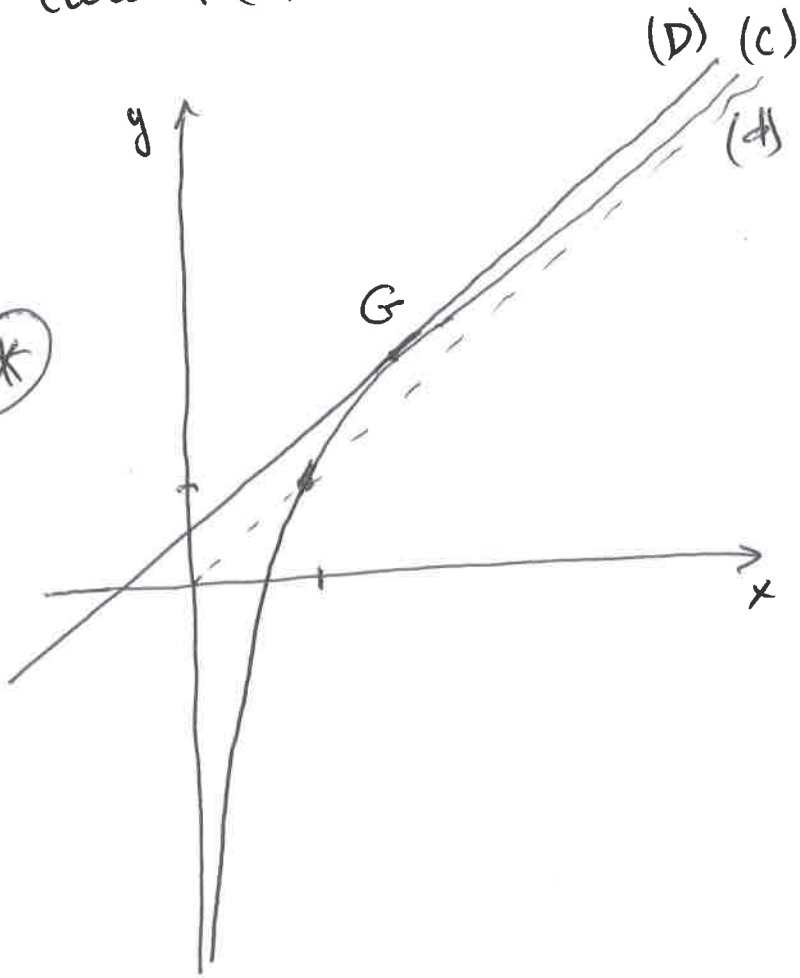
(4) f is continuous and strictly increases from

3 (*) $-\infty < 0$ to $+\infty > 0$ then $f(x)=0$ has a unique root

$f(0.75) \times f(0.76) = -0.00065 < 0$

3 (*) then $f(\alpha)=0$ where $0.75 < \alpha < 0.76$

3)



4 (*)