This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School

for Boys and Girls Loueizeh - Baabda

Stage Exam 02

08DEC2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	12E
Teacher :	Bishara Kordahi
Duration :	1 hour

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

Question I-(60 points)

Consider the function f defined over $\Re$ as $(C): f(x) = (1-x)\,e^x + 2$.

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$. Deduce an asymptote (d) to (C) .

b- Determine $\lim\limits_{x\to+\infty} f(x)$. Then , calculate $f(1)$ and $f(2)$.

2) a- Verify that $f'(x) = -xe^x$. Then , set-up a table of variations of f .

b- Prove that (C) admits a point of inflection I to be determined .

3) Draw (d) and (C) .

4) Denote by (Δ) the line $y = 2x$. Verify that $f(x) - 2x = (e^x + 2)(1-x)$.

Then , study according to the values of x , the relative positions of (C) and (Δ) .

5) Let g be the function given as $(G): g(x) = Ln[f(x) - 2]$.

a- Verify that its domain is $]-\infty, 1[$.

b- Is there a point on (G) where the tangent to (G) is parallel to (Δ) ? Justify .

Question II-(60 points)

A- Let g be the function defined through positive values of x as $g(x) = x^3 - 1 + 2Lnx$

1) Determine the limits of $g(x)$ at the boundaries of its domain .

2) Calculate $g'(x)$. Then , set-up a table of variations of $g(x)$.

3) Calculate $g(1)$ then deduce the sign of $g(x)$ according to the values of x .

B- Consider the function f defined as $(C): f(x) = x - \dfrac{Lnx}{x^2}$ over $]0, +\infty[$.

Let (L) be the line $y = x$.

1) Determine $\lim\limits_{x\to0} f(x)$ and deduce an asymptote to (C) .

2) a- Discuss , according to the values of x , the relative position of (C) and (L)

b- Determine $\lim\limits_{x\to+\infty} f(x)$ and prove that (L) is an asymptote to (C) .

3) a- Verify that $f'(x) = \dfrac{g(x)}{x^3}$. Then , set-up a table of variations of $f(x)$.

b- Determine the point E on (C) where the tangent (Δ) to (C) at E is parallel to (L)

c- Plot (L) and (C) .

I (C) : $f(x) = (1-x)e^x + 2$ $\quad D_f = \mathbb{R}$

1) (a) $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \left(\frac{1-x}{e^{-x}} + 2\right) = \lim_{x \to -\infty} \left(\frac{+1}{+e^{-x}} + 2\right) = 0 + 2 = 2$ 3

Then (d) : $y = 2$ HA 2

(b) $\lim_{x \to +\infty} f(x) = -\infty + 2 = -\infty$ 3

$f(1) = 2$, $f(2) = 2 - e^2 \approx -5.4$ 2

2) (a) $f'(x) = -e^x + e^x(1-x) = -xe^x$ 5

$f'(x) = 0$, $x = 0$ $\quad y = 3$

x	$-\infty$		0		$+\infty$
f'		$+$	0	$-$	
f	2	↗	max 3	↘	$-\infty$

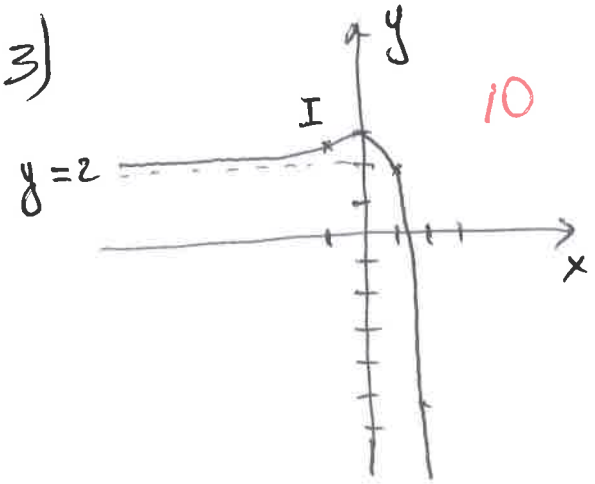
10

(b) $f''(x) = -\left[1 \cdot e^x + xe^x\right] = (-1-x)e^x$

$f''(x) > 0$, $x < -1$ $\quad f''(x) = 0$, $x = -1$ $\quad y = 2 + \frac{2}{e}$

$f''(x) < 0$, $x > -1$ 5 $\quad I\left(-1, 2 + \frac{2}{e}\right)$

3)



10

4) $(\Delta) : y = 2x$

$f(x) - 2x = (1-x)e^x + 2 - 2x$

$= (1-x)e^x + 2(1-x)$

$= (1-x)(e^x + 2) = g(x)$ 10

$g(x) = 0$, $x = 1$ (C) cuts (Δ) at $A(1, 2)$

$g(x) > 0$, $x < 1$ (C) above (Δ)

$g(x) < 0$, $x > 1$ (C) below (Δ)

5) $(G) : g(x) = \ln\left[f(x) - 2\right] = \ln\left[(1-x)e^x\right]$ 5

(a) $(1-x)e^x > 0$, $1 - x > 0$, $x < 1$ or $x \in]-\infty, 1[$

(b) $g'(x) = 2$, $\frac{-xe^x}{(1-x)e^x} = 2$, $-x = 2 - 2x$, $x = 2 \notin D_g$ 5

II A $g(x) = x^3 - 1 + 2\ln x$ $\quad D_g =]0, +\infty[$

1) $\lim_{x \to 0^+} g(x) = 0 - 1 - \infty = -\infty$ 2 , $\lim_{x \to +\infty} g(x) = +\infty - 1 + \infty = +\infty$ 3

2) $g'(x) = 3x^2 + \frac{2}{x} > 0$ 2

x	0		$+\infty$
g'		$+$	
g	$-\infty$	↗	$+\infty$

3

3) $g(1) = 1 - 1 + 0 = 0$ 2 $\quad g(x) \geq 0 \to x \geq 1$, $g(x) < 0 \to x < 1$ 3

B $(C) : f(x) = x - \frac{\ln x}{x^2}$ $\quad D_f =]0, +\infty[$ $\quad (L) : y = x$

1) $\lim_{x \to 0} f(x) = 0 + \frac{\infty}{0} = +\infty$ $\quad x = 0$ VA 5

2) (a) $h(x) = f(x) - x = \frac{-\ln x}{x^2}$ $\quad h(x) = 0, x = 1$ (C) cuts (L) at (1, 1) 5

$h(x) > 0, 0 < x < 1$ (C) above (L)

$h(x) < 0, x > 1$ (C) below (L)

(b) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left(x - \frac{\frac{1}{x}}{2x}\right) = \lim_{x \to +\infty} \left(x - \frac{1}{2x^2}\right) = +\infty - 0 = +\infty$ 2

$\lim_{x \to +\infty} \left[f(x) - x\right] = \lim_{x \to +\infty} \left(\frac{-1}{2x^2}\right) = 0^-$ 3 (L) asymptote to (C) as $x \to +\infty$

3) (a) $f'(x) = 1 - \frac{\frac{1}{x} \cdot x^2 - 2x\ln x}{x^4} = 1 - \frac{x(1 - 2\ln x)}{x^4 x^3} = \frac{x^3 - 1 + 2\ln x}{x^3}$

$f'(x) = \frac{g(x)}{x^3}$ 5

x	0		1		$+\infty$
f'	‖	$-$	0	$+$	
f	$+\infty$	↘	min $(1,1)$	↗	$+\infty$

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(b) $f'(x_E) = 1$, $\frac{x^3 - 1 + 2\ln x}{x^3} = 1$, $x^3 - 1 + 2\ln x = x^3$

$\ln x = \frac{1}{2}$ $\quad x_E = \sqrt{e}$ $\quad y_E = \sqrt{e} - \frac{1}{2e}$ 5

(c)

