This is a Class 1 exam

Lebanon Evangelical School
for Boys and Girls
Loueizeh - Baabda

Stage Exam 06

13JUN2024 and 14JUN2024

Do NOT write in this box. Your MARK will be placed here.

Subject :	Maths
Class :	11SE
Teacher :	Mr. George El Haber
Duration :	1 hour

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

I. A survey was conducted in a super store to study the monthly turnover in the number of clients. **[18 marks]**

- During the first month, 5000 clients made their purchases from the store.
- Each month 30% of the clients of the preceding month stopped to make their purchases from this store and 1800 new clients joined the remaining clients of this store.

For all natural non-zero integers n, denote by U_n the number of clients that make their purchases during the nth month, where $U_1 = 5000$

1) a- Verify that $U_2 = 5300$ and calculate U_3.
   b- Show that the sequence (U_n) is neither arithmetic nor geometric.

2) Justify the relation $U_{n+1} = 0.7\,U_n + 1800$

3) (V_n) is the sequence defined for all natural non-zero integers n by $V_n = 6000 - U_n$
   a- Prove that the sequence (V_n) is a geometric sequence whose common ratio and first term are to be determined.
   b- Calculate V_n in terms of n and deduce U_n in terms of n.

II. An urn contains 5 red balls, 2 black balls and 3 green balls. **[12 marks]**

3 balls are drawn randomly and successively without replacement.

a- What is the probability that the 3 balls have the same color?

b- What is the probability that the 3 balls have different colors?

c- What is the probability that the 3 balls do not have the same color?

d- What is the probability that the 3 balls are red, red, and green in this order?

III. A and B are two events such that: **[10 marks]**

$$P(\bar{A}) = \frac{2}{7}, \qquad P(A \cup B) = \frac{6}{7}, \qquad P(\bar{B}) = \frac{4}{7}$$

a- Calculate $P(A)$ and $P(A \cap B)$

b- Calculate $P(A \cap \bar{B})$ and $P(\bar{A} \cap \bar{B})$

----------George El Haber----------

Maths 11SE SE06 Answerkey

I. $U_1 = 5000$ decreases 30% and 1800 new

1) a- $U_2 = \left(1 - \frac{30}{100}\right) U_1 + 1800$

$= 0.7(5000) + 1800$

$= 5300$

$U_3 = \left(1 - \frac{30}{100}\right) U_2 + 1800$

$= 0.7(5300) + 1800$

$= 5510$

b- $U_2 - U_1 = 5300 - 5000 = 300$

$U_3 - U_2 = 5510 - 5300 = 210$

So (U_n) is not arithmetic

$\frac{U_3}{U_2} = 1.039$ and $\frac{U_2}{U_1} = 1.06$

so (U_n) is not Geometric

2) $U_{n+1} = \left(1 - \frac{30}{100}\right) U_n + 1800$

$= 0.7 U_n + 1800$ Verified

3) $V_n = 6000 - U_n$

a- $V_{n+1} = 6000 - U_{n+1}$

$= 6000 - 0.7U_n - 1800$

$= 4200 - 0.7U_n$

$$\frac{V_{n+1}}{V_n} = \frac{4200 - 0.7U_n}{6000 - U_n}$$

$$= \frac{0.7(6000 - U_n)}{6000 - U_n} = 0.7 \quad \text{constant}$$

Then V_n is a G.S. with common ratio $= 0.7$

$V_n = 6000 - U_n$

so, $V_1 = 6000 - U_1$

$= 6000 - 5000$

$= 1000$

b- General term

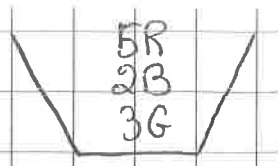
$V_n = V_1 \times r^{n-1}$

$= 1000 \times 0.7^{n-1}$

Then, $U_n = 6000 - V_n$

$U_n = 6000 - 1000(0.7)^{n-1}$

III.



a- $P(\text{Same color}) = P(RRR \cup GGG)$

$$= \left(\frac{5}{10} \times \frac{4}{9} \times \frac{3}{8}\right) + \left(\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}\right)$$

$$= \frac{11}{120}$$

b- $P(\text{Different colors}) = P(RBG) \times 3!$

$= \left(\frac{5}{10} \times \frac{2}{9} \times \frac{3}{8}\right) \times 3!$

$= \frac{1}{4}$

c- $P(\text{Not the Same}) = 1 - P(\text{Same})$

$= 1 - \frac{11}{120}$

$= \frac{109}{120}$

d- $P(RRG) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8}$

$= \frac{60}{720} = \frac{1}{12}$

IV.

a- $P(A) = 1 - P(\bar{A})$

$= 1 - \frac{2}{7}$

$= \frac{5}{7}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{5}{7} + \frac{4}{7} - \frac{6}{7}$

$= \frac{3}{7}$

b- $P(A) = P(A \cap B) + P(A \cap \bar{B})$

$P(A \cap \bar{B}) = P(A) - P(A \cap B) = \frac{5}{7} - \frac{2}{7} = \frac{3}{7}$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - \frac{6}{7} = \frac{1}{7}$

Or

$P(\bar{A} \cap \bar{B}) = P(\bar{B}) - P(A \cap \bar{B})$

$= \frac{4}{7} - \frac{3}{7} = \frac{1}{7}$