This is a Class 1 exam

Lebanon Evangelical School for Boys and Girls

Lebanon Evangelical School

for Boys and Girls Loueizeh - Baabda

Stage Exam 05

12MAY2023

Do NOT write in this box. Your MARK will be placed here.

Subject :	A Level : Maths M2Mechanics
Class :	12A
Teacher :	Bishara Kordahi
Duration :	1 hour 30 minutes

Your name :		*Time taken to complete exam*
Your Class :		
Exam Room :		
How many sheets, including cover sheet :		

- Do not open this paper until the teacher in charge instructs you to do so.
- The most important thing to remember in taking an exam is NEVER PANIC.
- If you are caught cheating in this exam, you will be awarded zero for ALL exams that are taken on this day.

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1. A motorcyclist and his motorcycle have a combined mass of 480 kg.

The motorcyclist drives down a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{12}$, with the engine of the motorcycle working at 3.5 kW. The motorcycle is moving at a constant speed of $V\,\text{m s}^{-1}$. The resistance to the motion of the motorcycle is modelled as a constant force with magnitude $20V$ newtons.

Find the value of V.

(5)

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2. A particle P of mass 1.5 kg moves under the action of a single force $\mathbf{F}$ newtons.

At time t seconds, $t \geqslant 0$, P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (5t^2 - t^3)\mathbf{i} + (2t^3 - 8t)\mathbf{j}$$

(a) Find $\mathbf{F}$ when $t = 2$

(4)

At time $t = 0$, P is at the origin O.

(b) Find the position vector of P relative to O at the instant when P is moving in the direction of the vector $\mathbf{j}$

(4)

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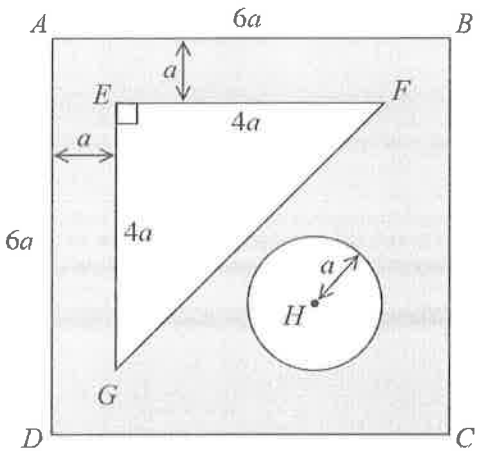


Figure 1

The uniform lamina $ABCD$ is a square of side $6a$. The template T, shown shaded in Figure 1, is formed by removing the right-angled triangle EFG and the circle, centre H and radius a, from the square lamina.

Triangle EFG has $EF = EG = 4a$, with EF parallel to AB and EG parallel to AD. The distance between AB and EF is a and the distance between AD and EG is a.

The point H lies on AC and the distance of H from BC is $2a$.

(a) Show that the centre of mass of T is a distance $\dfrac{4(67-3\pi)}{3(28-\pi)}a$ from AD.

(5)

The template T is suspended from the ceiling by two light inextensible vertical strings. One string is attached to T at A and the other string is attached to T at B so that T hangs in equilibrium with AB horizontal.

The weight of T is W. The tension in the string attached to T at B is kW, where k is a constant.

(b) Find the value of k, giving your answer to 2 decimal places.

(3)

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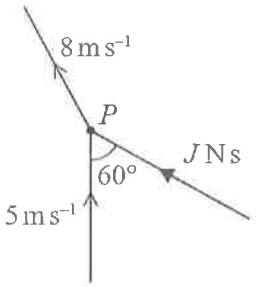


Figure 2

A particle P of mass 0.3 kg is moving with speed $5\,\text{m s}^{-1}$ along a straight line on a smooth horizontal plane. The particle receives a horizontal impulse of magnitude J Ns. The speed of P immediately after receiving the impulse is $8\,\text{m s}^{-1}$. The angle between the direction of motion of P before it receives the impulse and the direction of the impulse is 60°, as shown in Figure 2.

Find the value of J.

(6)

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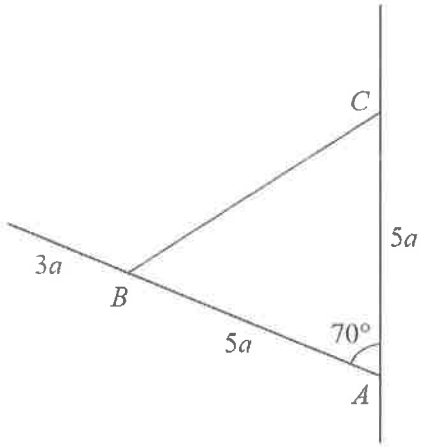


Figure 3

A uniform rod, of length $8a$ and mass M, has one end freely hinged to a fixed point A on a vertical wall. One end of a light inextensible string is attached to the rod at the point B, where $AB = 5a$. The other end of the string is attached to the wall at the point C, where $AC = 5a$ and C is vertically above A. The rod rests in equilibrium in a vertical plane perpendicular to the wall with angle $BAC = 70°$, as shown in Figure 3.

(a) Find, in terms of M and g, the tension in the string.

(3)

The magnitude of the force acting on the rod at A is λMg, where λ is a constant.

(b) Find, to 2 significant figures, the value of λ.

(6)

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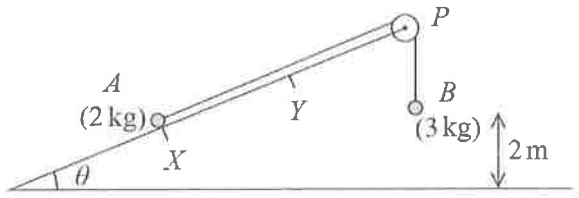


Figure 4

Two particles, A and B, of mass 2 kg and 3 kg respectively, are connected by a light inextensible string. Particle A is held at rest at the point X on a fixed rough ramp that is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The string passes over a small smooth pulley P that is fixed at the top of the ramp. Particle B hangs vertically below P, 2 m above the ground, as shown in Figure 4.

The particles are released from rest with the string taut so that A moves up the ramp and the section of the string from A to P is parallel to a line of greatest slope of the ramp. The coefficient of friction between A and the ramp is $\frac{3}{8}$

Air resistance is ignored.

(a) Find the potential energy lost by the system as A moves 2 m up the ramp.

(3)

(b) Find the work done against friction as A moves 2 m up the ramp.

(4)

When B hits the ground, B is brought to rest by the impact and does not rebound and A continues to move up the ramp.

(c) Use the work-energy principle to find the speed of B at the instant before it hits the ground.

(4)

Particle A comes to instantaneous rest at the point Y on the ramp, where $XY = (2 + d)$ m.

(d) Use the work-energy principle to find the value of d.

(4)

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7. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

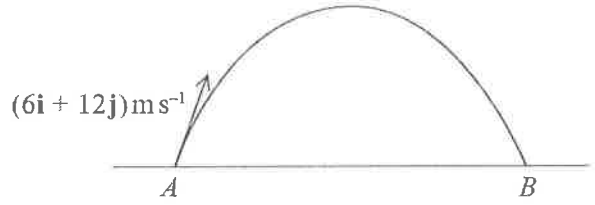


Figure 5

A small ball is projected with velocity $(6\mathbf{i} + 12\mathbf{j})\,\text{m s}^{-1}$ from a fixed point A on horizontal ground. The ball hits the ground at the point B, as shown in Figure 5. The motion of the ball is modelled as a particle moving freely under gravity.

(a) Find the distance AB.

(4)

When the height of the ball above the ground is more than h metres, the speed of the ball is less than $10\,\text{m s}^{-1}$

(b) Find the smallest possible value of h.

(4)

When the ball is at the point C on its path, the direction of motion of the ball is perpendicular to the direction of motion of the ball at the instant before it hits the ground at B.

(c) Find, in terms of **i** and **j**, the velocity of the ball when it is at C.

(3)

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8. Particles A, B and C, of masses $2m$, m and $3m$ respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particle A is projected towards particle B with speed $2u$ and collides directly with B.

The coefficient of restitution between each pair of particles is e.

(a) (i) Show that the speed of B immediately after the collision with A is $\frac{4}{3}u(1+e)$

(ii) Find the speed of A immediately after the collision with B.

(7)

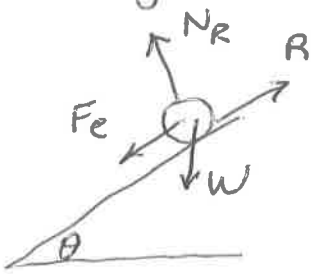
At the instant when A collides with B, particle C is projected with speed u towards B so that B and C collide directly.

(b) Show that there will be a second collision between A and B.

(6)

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M2 Mock exam correction — May 2023



(1) $M = 480$ $\quad F_e + W\sin\theta - R = M.a = 0$

$\sin\theta = \frac{1}{12}$ $\quad \frac{3500}{V} + \frac{480(9.8)}{12} - 20V = 0$

$\frac{3500}{V} + 392 - 20V = 0$, $\quad 20V^2 - 392V - 3500 = 0$

$V = \frac{392 \pm \sqrt{392^2 + 4(20)(3500)}}{2(20)}$ $\quad V_1 = 26.3\ ms^{-1}$, $V_2 = -6.7 < 0$

5 $\boxed{V = 26.3}$

---

(2) $M = 1.5$ $\quad v = (5t^2 - t^3)i + (2t^3 - 8t)j \quad t \geq 0$

a) $F = Ma = M\dot{v} = 1.5\left[(10t - 3t^2)i + (6t^2 - 8)j\right]$

$t = 2$ $\quad \boxed{F_2 = (12i + 24j)\,N}$ 4

b) $t = 0$, $r = 0$

Let's first find the time t when P is moving in the direction of j. Then $5t^2 - t^3 = 0$

$t^2(5 - t) = 0 \Rightarrow t = 0$ rejected, $\boxed{t = 5}$

$r = \int v\,dt = \left(\frac{5t^3}{3} - \frac{t^4}{4} + A\right)i + \left(\frac{2t^4}{4} - \frac{8t^2}{2} + B\right)j$

$r = \left(\frac{5t^3}{3} - \frac{t^4}{4} + A\right)i + \left(\frac{t^4}{2} - 4t^2 + B\right)j$

→ $A = 0$ and $B = 0$. Then:

$r = \left(\frac{5t^3}{3} - \frac{t^4}{4}\right)i + \left(\frac{t^4}{2} - 4t^2\right)j$

At $t = 5$ $\quad r_5 = \left(\frac{625}{3} - \frac{625}{4}\right)i + \left(\frac{625}{2} - 100\right)j$ 4

$\boxed{r_5 = \left(\frac{625}{12}i + \frac{425}{2}j\right)m}$ 1/6

(3) a) Choosing the origin of axes at D

1: ABCD
2: EFG
3: circle

E(a, 5a) F(5a, 5a)
G(a, a)
H(4a, 2a)

$$\bar{x} = \frac{x_1A_1 + x_2A_2 + x_3A_3}{A_1 + A_2 + A_3}$$

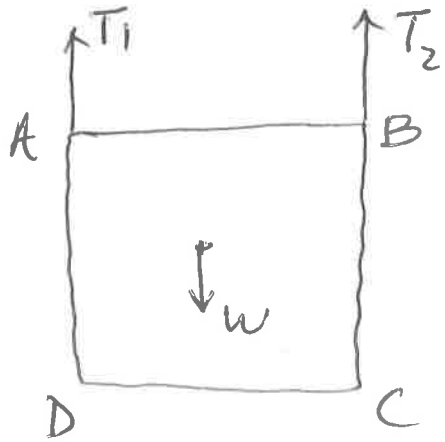
$$\bar{x} = \frac{3a(36a^2) + \frac{7a}{3}\left(-\frac{16a^2}{2}\right) + 4a\left(-\pi \cdot a^2\right)}{36a^2 - 8a^2 - \pi a^2}$$

$$\bar{x} = \frac{108 - \frac{56}{3} - 4\pi}{28 - \pi} \cdot a \Rightarrow \bar{x} = \frac{324 - 56 - 12\pi}{3(28-\pi)} = \frac{268 - 12\pi}{3(28-\pi)}$$

5

Hence $\boxed{\bar{x} = \frac{4(67-3\pi)}{3(28-\pi)} \cdot a}$ = distance of G' from AD.

b) Moments about A:



$$T_2(6a) = W(\bar{x})$$

$$(K \cdot W)(6a) = W\left[\frac{4(67-3\pi)}{3(28-\pi)}\right]a$$

3

$$K = \frac{4(67-3\pi)}{18(28-\pi)} \rightarrow \boxed{K \approx 0.51}$$

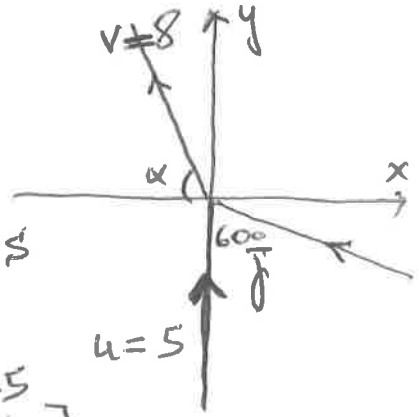
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(4) $M = 0.3\,kg \quad u = (5j)\ ms^{-1}$

$$v = (-8\cos\alpha)i + (8\sin\alpha)j\ ms^{-1}$$

Impulse $= (-J\sin 60)i + (J\cos 60)j\ N.s$

Impulse $= M(v - u)$



$$-\frac{J\sqrt{3}}{2}i + \frac{J}{2}j = 0.3\left[(-8\cos\alpha)i + (8\sin\alpha - 5)j\right]$$

then $-\frac{J\sqrt{3}}{2} = -2.4\cos\alpha$ and $\frac{J}{2} = 2.4\sin\alpha - 1.5$

$$\sin\alpha = \frac{J/2 + 1.5}{2.4} \qquad \cos\alpha = \frac{J\sqrt{3}}{4.8}$$

$$\sin\alpha = \frac{J + 3}{4.8}$$

$\cos^2\alpha + \sin^2\alpha = 1 \rightarrow \left(\frac{J\sqrt{3}}{4.8}\right)^2 + \left(\frac{J+3}{4.8}\right)^2 = 1$

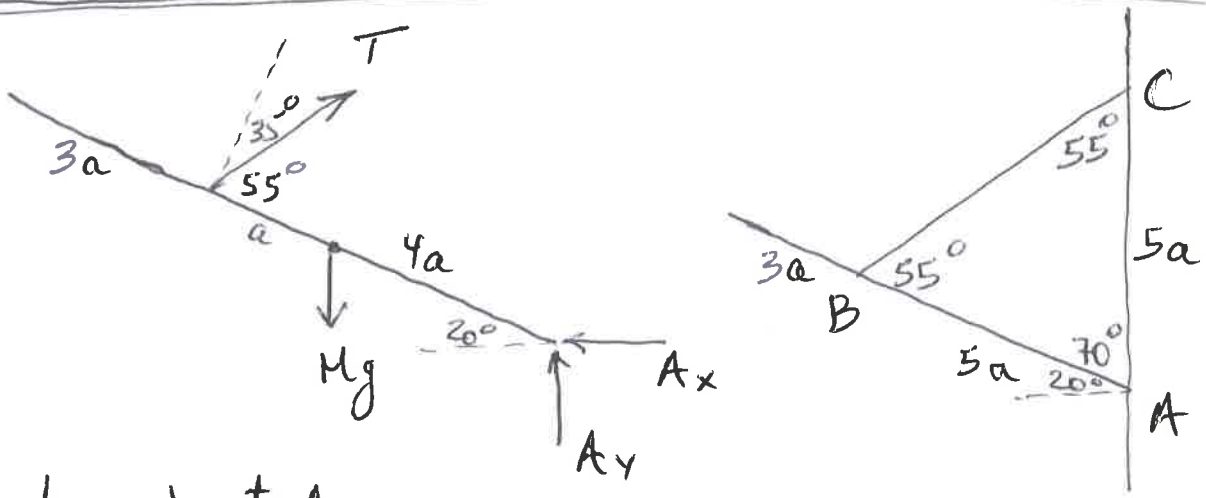
$3J^2 + 6J + 9 + J^2 = 4.8^2 = 23.04$

$4J^2 + 6J - 14.04 = 0 \qquad J = \frac{-6 \pm \sqrt{(6)^2 + 16(14.04)}}{8}$

$J_1 = 1.27\ N.s$ 6 $\qquad J_2 = -2.77 < 0$ rejected

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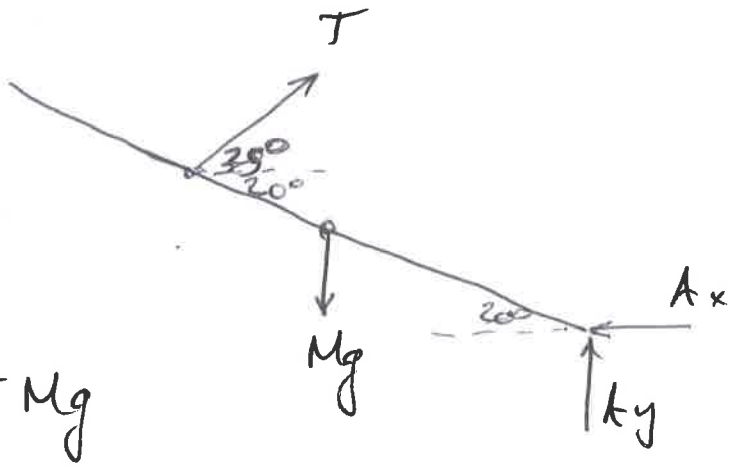
(5)



a) Moments about A:

$Mg(4a\cos 20°) = (T\cos 35°)(5a)$, $T = \frac{4\cos 20}{5\cos 35} Mg$

$T = 0.92\ Mg$ 3



b) $A_x = T\cos 35°$

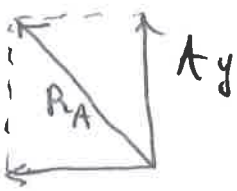
$A_y + T\sin 35° = Mg$

Then $A_x = 0.75\ Mg$ 1

$A_y = Mg - \frac{4\cos 20}{5\cos 35}\cdot \sin 35\ Mg$

$A_y = 0.47\ Mg$ 2

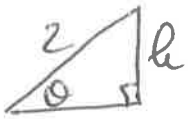
$R_A = \lambda Mg$ (given)

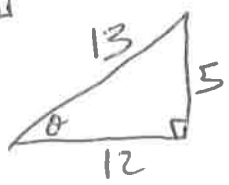


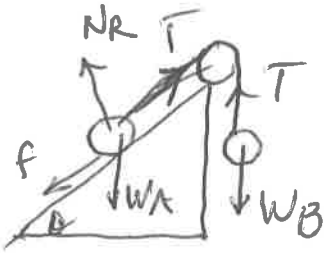
$R_A^2 = A_x^2 + A_y^2$, $\lambda^2 (Mg)^2 = (0.75^2 + 0.47^2)(Mg)^2$

$\lambda = 0.89$ 3

(6) $M_A = 2kg$ $M_B = 3kg$ $\mu = \frac{3}{8}$







a) $PE_{\text{gained by A}} = M_A \cdot g \cdot h_A = 2(9.8)(2\sin\theta) = 15.08\,J$

$PE_{\text{lost by B}} = M_B \cdot g \cdot h_B = 3(9.8)(2) = 58.8\,J$

$PE_{\text{lost by system}} = 58.8 - 15.08 = \boxed{43.72\,J}$ 3

b) $f = \mu N_R = \mu M_A g\cos\theta = \frac{3}{8}(2)(9.8)\left(\frac{12}{13}\right) = \frac{441}{65}\,N$

Work done against friction $= \frac{441}{65} \times 2 = \boxed{13.57\,J}$ 4

c) $\Delta KE_{\text{system}} + \Delta PE_{\text{system}} = \text{Work done by friction}$

$\frac{1}{2}(M_A+M_B)(v^2 - u^2) - 43.72 = -13.57$ ($u = 0$)

$\frac{1}{2}(5)v^2 = 30.15$, $v^2 = 12.06 \rightarrow v = 3.47\,ms^{-1}$

$\boxed{v_A = v_B = 3.47\,ms^{-1}}$ 4

d) $\Delta KE_A + \Delta PE_A = \text{Work done by friction}$

$\frac{1}{2}M_A(0^2 - 3.47^2) + M_A(9.8)(d\sin\theta) = -\frac{441}{65}d$

$-\frac{1}{2}(2)(12.06) + \frac{2(9.8)d(5)}{13} = -\frac{441}{65}d$ 4

$14.32d = 12.06$ $\boxed{d = 0.84\,m}$

(7) $u = 6i + 12j$

a) $AB = \text{Range} = \dfrac{u^2 \sin 2\alpha}{g} = \dfrac{2u\sin\alpha \cdot u\cos\alpha}{g}$

$AB = \dfrac{2(12)(6)}{9.8} \rightarrow \boxed{AB = 14.7\text{m}}$ 4

b) Conservation of mechanical energy:

$KE_{lost} = PE_{gained}$

$\frac{1}{2}M(u^2 - v^2) = Mgh$, $h = \dfrac{u^2 - v^2}{2g}$

$h = \dfrac{6^2 + 12^2 - 10^2}{2(9.8)} \rightarrow \boxed{h_{least} = 4.1\text{ m}}$ 4

c) Due to symmetry $v_B = (6i - 12j)\,ms^{-1}$

$v_C \perp v_B$ with $(v_C)_x = (v_B)_x = 6i$

Then $v_C = 6i + vj$ but gradient$_{v_B}$ × gradient$_{v_C}$ = −1

Then $\dfrac{-12}{6} \cdot \dfrac{v}{6} = -1$, $-12v = -36$, $v = 3$

Hence $\boxed{v_C = (6i + 3j)\,ms^{-1}}$ 3

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(8)

A	B	C
$M_A = 2m$	$M_B = m$	$M_C = 3m$
$u_A = 2u \rightarrow$	$u_B = 0$	$u_C = u \leftarrow$
$v_A = \rightarrow$	$v_B = \rightarrow$	$v_C = \rightarrow$
	$v_B' = \leftarrow$	

a) $m_A u_A + m_B u_B^{\,0} = m_A v_A + m_B v_B$

$4mu = 2mv_A + mv_B$, $2v_A + v_B = 4u$

$e = \frac{v_B - v_A}{u_A - u_B} = \frac{v_B - v_A}{2u - 0}$, $v_A - v_B = 2eu$

$3v_A = 4u - 2eu$

$v_A = \frac{2u(2-e)}{3}$ → since $e \leq 1$

$v_B = v_A + 2eu = \frac{4u - 2eu}{3} + 2eu = \frac{4u - 2eu + 6eu}{3}$

then $v_B = \frac{4u + 4eu}{3} = \frac{4u(1+e)}{3}$ → QED

b) $m_B v_B + m_C u_C = m_B v_B' + m_C v_C$

$\frac{4mu(1+e)}{3} - 3mu = -mv_B' + 3mv_C$

$\frac{4u + 4eu}{3} - 3u = -v_B' + 3v_C = \frac{4u + 4eu - 9u}{3}$

$-v_B' + 3v_C = \frac{4eu - 5u}{3}$ $\qquad e = \frac{v_C - v_B'}{v_B - u_C}$

$e = \frac{v_C + v_B'}{\frac{4u+4eu}{3} + u} = \frac{v_C + v_B'}{\frac{4u+4eu+3u}{3}}$, $v_C + v_B' = \frac{(7u+4eu)e}{3}$

$\begin{cases} v_B' - 3v_C = \frac{5u - 4eu}{3} \\ 3v_B' + 3v_C = 7eu + 4e^2u \end{cases}$

$4v_B' = \frac{5u - 4eu + 21eu + 12e^2u}{3}$ 6

$v_B' = \frac{5u + 17eu + 12e^2u}{12}$ ← since $v_B' > 0$

hence B re-collides with A since

$v_A \rightarrow \qquad v_B' \leftarrow$